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Between cointegration and multicointegration: Modelling time series dynamics by cumulative error correction models☆

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ABSTRACT

This study proposes a cumulative error correction model where the summing weights follow a geometrically decreasing function of prior deviations from the equilibrium and are estimated from the data. It is shown that this approach nests both the traditional error correction model – where no weight is given to deviations from the steady state prior to the most recent period – and the error correction model based on the idea of multicointegration.

The form of accumulation presented here does not change the order of integration of the series, as is the case in the multicointegration approach of Granger and Lee (1989). Furthermore, it is very parsimonious as only one or two parameters more have to be estimated. The assumption of geometrically decreasing weights can be tested by estimating the model in its unrestricted form.

Based on this new model type, the relationship between private consumption and real disposable income of private households in the US is estimated. The short-term forces which set off the most recent period's deviations are much smaller than would be suggested by a VEC and a conventional single equation ECM, and the income elasticity is lower as well. The proposed model outperforms the other two with respect to its forecasting power.

1. Introduction

Following the seminal work of Granger and Newbold (1974), the idea of cointegration-based modelling has become very popular – not only because the authors have shown that regressing integrated time series on each other can cause spurious results, but also because cointegrated variables possess a so-called Error Correction Model (ECM) representation.[1] Furthermore, such models frequently offer a reasonable economic interpretation.

For this reason ECMs are still an important workhorse for modelling and forecasting private consumption. A prominent pioneering work in this direction was Davidson et al. (1978). Whereas the discussion of the correct long-run relationship between private consumption expenditures generally aims at discriminating between different theories, forecasting short-run consumption dynamics is also of great value for guiding economic policy decisions.

When short-run dynamics are modelled by an ECM the change in current endogenous variables depends not only on the change of a set of explaining variables but also on the discrepancy between the observed endogenous variable and its long-run relationship of the most recent period (t-1). The reasoning behind this is that the idea of cointegration demands that the observed variable returns to its steady state across some time horizon. While this is a perfectly understandable condition, it is not automatically obvious why conventional ECMs consider only the most recent imbalance in estimating the attracting forces of the following period. Thus, when estimating a consumption function, the mismatch in the cointegrating relationship between income and consumption can be interpreted as saving activities. So in the case of a conventional ECM only the savings of the most recent period of the past exert an influence on consumption decisions during the current period. All other savings of periods further in the past are not considered.[2] This implies a rather strong assumption, especially for high-frequency time series (e.g. quarterly, monthly, weekly or even more frequent).

Granger and Lee (1989) tried to account for this shortcoming of conventional ECMs and developed the concept of multicointegration where all disequilibria that had occurred in the past are summed up in order to form a further cointegrating relationship. However, this not only complicates the estimation process considerably but, by assuming that all deviations of the past – even the most distant – play the same role in the current adjustment process towards the steady

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[1] This is called the Granger Representation Theorem (Engle and Granger, 1987).

[2] For this reason some authors consider wealth as an additional explanatory variable in their model specification.

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state, overshoots the mark. For that reason this model type did not become very popular. As far as I know, only two examples exist where US private consumption was modelled using multicointegration models. Contrary to Lee (1996) who found no evidence for multicointegration between US consumption and income, Siliverstovs (2003) did so for the period of 1953–1984.

This paper proposes a new type of ECM which is located between conventional ECMs and the concept of multicointegration. While the first approach ignores and the second overemphasises disequilibria of periods located further in the past than the most recent period in their error adjusting process, this approach suggests a gradual decrease of weights for more distant imbalances. The speed of the gradual decrease is estimated from the data. Only the structure of the decrease is assumed to follow a geometric process. This new approach has several attractive advantages. Firstly, contrary to the concept of multicointegration the estimation can be kept simple as no I(2) variables have to be considered. Secondly, this model type nests conventional ECMs as well as multicointegration ECMs. Since partial-adjustment models – used for modelling a sluggish reversal to equilibrium – can be represented as a conventional ECM with certain parameter restrictions, they can be considered as nested as well. The proposed model can be estimated in an unrestricted form, which allows testing the assumption of geometrically decreasing weights at the same time, as well as in a restricted form.

This paper is organised as follows: Section 2 presents the so-called cumulative error correction model (cumECM) together with its properties and differences to other ECMs. Section 3 provides as an empirical example a model for estimating private consumption in the US. There the steady state is estimated by different methods and short-run dynamics are compared between the conventional ECM and the cumECM in its restricted and unrestricted forms. Section 4 compares the forecasting power between the different approaches and shows that the cumECM considerably outperforms the conventional specification. Section 5 summarises the conclusions.

2. The model

In several economic applications the assumption that only a deviation from the steady state of the most recent period influences short-term movements has been found to be inadequate. Past disequilibria between income and consumption can lead to an accumulation of a wealth stock which determines future consumption behaviour. Similarly, the mismatch between production and sales leads to accumulated inventories, and between money demand and income to a stock of accumulated money. As the usual ECMs in this case would result in a misspecification error,[3] Granger and Lee (1989) developed the idea of multicointegration. Apart from the cointegrating relationship between the flows (first-level cointegration) there may be another one coming from stocks. This they called second-level cointegration. Whereas usually the number of cointegrating relationships among n variables is at most n–1, with multicointegration it may be n as well.

Suppose that y_t and x_t are both I(1) and are cointegrated CI(1,1) so that

$$z_t = y_t - \varphi x_t \tag{1}$$

is I(0). Eq. (1) is the so-called first-level cointegration relationship. The authors further propose that past deviations from the steady state accumulate to a stock variable $s_t = \sum_{i=1}^{t} z_t$. If s_t cointegrates with either x_t or y_t, we get another cointegration relationship (called second-level cointegration) so that s_t – κy_t forms a stationary relationship again:

$$s_t - \kappa y_t = \sum_{j=1}^{t} y_t - \varphi \sum_{j=1}^{t} x_t - \kappa y_t, \tag{2}$$

with s_t being I(1) as it stems from summing I(0) stock changes, and Σy_t and Σx_t being I(2) as both are summed I(1) variables.[4] Granger and Lee (1989) solved the estimation problem by a two-step method as is typically used for CI(1,1) variables. In a first step they estimated the first-level cointegration relationship. The residuals as deviations from the steady state were summed up and in a second stage were regressed on the cumulated variables (summed y_t) for estimating the second-order integrating relationship.

However, Engsted et al. (1997) have shown that in the case of a two-step method the first cointegrating relationship (of flows) must not be estimated because the test statistics of the second one would have a different limiting distribution compared to normal settings. Furthermore, for I(2)-based models the usual asymptotic χ^2 inference is invalid and Johansen (2006) pointed out that it can be used only if a multicointegration relationship is assumed with properties which are hardly ever met in reality. Engsted et al. (1997) proposed a single equation method where both forms of integration are tested together, and supplied tables with critical test levels.

The model presented here does not – like in the multicointegration approach – sum up past deviations s_t with equal weights. Instead another weighting scheme is chosen which retains the series I(0) even after accumulation. Therefore, no second cointegrating relationship is necessary ($\kappa = 0$). But in contrast to the conventional ECM it does not give a weight of zero to deviations prior to (t-1). It assumes instead that the weights are decreasing the further they are located in the past, along a geometric process. Therefore, more distant deviations from the steady state are less important for explaining short-run dynamics than the ones which happened in the more recent past. A typical application in economics is that past investments form today's capital stock. Depreciation reduces past investments so that their contribution to the capital stock is decreasing from period to period. Other forces which reduce the impact of past deviations can be surprise inflation or changes in asset prices (house prices and securities), etc.

The decrease of weights seems to be a realistic assumption (at least as realistic as assuming weights of zero or one) in several cases and has the advantage that the compilation of I(2) variables can be avoided. Furthermore, the estimation is very parsimonious as will be shown below.

Instead of giving each deviation of the past the constant summation weight of one, as Granger and Lee (1989) did, our weighting parameter is estimated from the data and is supposed to be smaller than one.[5]Based upon these considerations, it is proposed to reformulate a conventional ECM

$$\Delta y_t = \beta \xi_{t-1} + \gamma \Delta x_t + c + u_t \tag{3}$$

[3] See Engsted and Johansen (1997) or Lee (1992).

[4] In the case of multicointegration the corresponding ECM considers adjustment mechanisms for the stock as well as the flow variables with $\Delta x_t = c + \alpha_1(s_{t-1} - \kappa y_{t-1}) + \alpha_2 \xi_{t-1} + lagged(\Delta x_t, \Delta y_t) + u_t$, with ξ_{t-1} being the deviation estimated from the first-level cointegrating relationship.

[5] The assumption of a retention rate $\lambda < 1$ is important insofar as otherwise the summation would result in an I(1) variable as in the case of multicointegration. In this case our model would be misspecified since a possibly existing second-level cointegrating relationship is not explicitly considered in our ECM.