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Variable deterioration and demand—An inventory model

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ABSTRACT

This paper deals with an inventory model for deteriorating items with inventory dependent demand function. Most of the inventory models are considered with constant rate of deterioration. In this article, we consider time varying deterioration rate. Based on the demand and inventory, the model is considered with three possible cases. We establish the necessary and sufficient conditions for each case to show the existence and uniqueness of the optimal solution. Further, a simple solution algorithm has proposed to obtain the optimal replenishment cycle time and ordering quantity such that the total profit per unit time is maximized. Finally, some numerical examples, sensitivity analysis and graphical representations are provided to illustrate the practical usages of the proposed method.

1. Introduction

It is well known to all that to attract customers, the retailers have to store wide range of items in stock. Silver and Peterson (1985) explained the effect of inventory displayed on the sales at the retail level. An EOQ (economic order quantity) model with consumption rate to minimize the cost with initial stock dependent demand was developed by Gupta and Vrat (1986). But, this assumption was very much restrictive. This restriction was removed by Baker and Urban (1988) by assuming that the demand rate as a function of the instantaneous stock level at any instant of time. Padmanabhan and Vrat (1995) discussed an inventory model in which the backlogging rate was dependent on the total number of customers. Chung et al. (2000) analyzed a stock-dependent inventory system where Newton–Raphson method was used to find the optimal solutions of the profit functions. Cárdenas-Barrón (2001) presented an inventory model with shortage by an algebraical approach. An idea of stock-dependent and time-varying demand pattern for deteriorating items over a finite time planning horizon was developed by Balkhi and Benkherouf (2004). Teng et al. (2005) developed an inventory model for deteriorating items with power-form stock-dependent demand. Chang et al. (2006) modified Balkhi and Benkherouf's (2004) model by introducing profitable building up inventory. Sana and Chaudhuri (2006) derived a model on a volume-flexible stock-dependent demand. Wu et al. (2006) represented an optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. An inventory model with stock-level dependent demand rate and variable holding cost was addressed by Alfares (2007). Chung and Wee (2007) discussed the scheduling and replenishment plan for an integrated deteriorating inventory model with stock-dependent selling rate. Most recently, in this direction, some notable researches were done by Goyal and Chang (2009), Yang et al. (2010), Cárdenas-Barrón (2011), Sarkar et al. (2010a, 2010b), Sarkar (2012a) and others. Recently Sarkar and Sarkar (2013) explained an improved inventory model with partial backlogging, time varying deterioration and stock-dependent demand.

A numerous number of researchers have investigated on inventory models with constant demand rate or time varying demand patterns. A few of the researchers like Barbosa and Friedman (1978), Datta and Pal (1988, 1990), Urban (1992), Urban and Baker (1997), Ray et al. (1998), and others have considered the demand of the items as power demand pattern. Yang et al. (2002) extended Barbosa and Friedman's (1978) model with shortages. An inventory model with power demand pattern and backorders in the one-warehouse *N*-retailer problem was developed by Abdul-Jalbar et al. (2009).

In daily life, the deterioration of items becomes a common factor. Generally, we define deterioration as decay or damage of items, such as fruits, foods, vegetables, etc. Highly volatile liquids like alcohol, turpentine, gasoline, radioactive materials, etc., deteriorate due to evaporation while kept in store.

Ghare and Schrader (1963) proposed an EOQ model for exponentially deteriorating items. Later, Covert and Philip (1973) extended that model assuming Weibull's distributed deterioration rate. An inventory model with three parameter Weibull's distribution rate was developed by Philip (1974). Later, an inventory model for deteriorating items with time-proportional demand without shortage was discussed by Dave and Patel (1981). Further, that model was extended by Sachan (1984) by using shortages. Since then, many researchers developed their excellent works in this field like Goyal (1987), Raafat (1991), Goswami and Chaudhuri (1992), Wee (1995), Hariga (1996), Wee and Law (1999), and Goyal and Giri (2001). Manna and Chaudhuri

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(2006) discussed an EOQ model for deteriorating items with both time-dependent demand and deterioration. Chung and Wee (2007) developed a scheduling and replenishment plan for an integrated deteriorating inventory model with stock dependent demand. An inventory model with ramp type demand and Weibull's deterioration rate, partial backlogging of unsatisfied demand, was presented by Skouri et al. (2009). Sana (2010) extended an EOQ model assuming optimal selling price and lotsize with time varying deterioration and partial backlogging. In that model, the deterioration function was considered as functional form of time. Hsu et al. (2010) presented a deteriorating inventory policy on the investment by the retailers under the preservation technology to reduce the rate of deterioration. An EOQ model for deteriorating items with planned backorder level was developed by Widyadana et al. (2011). An investigation on short life-cycle deteriorating product remanufacturing in a green supply chain inventory control system was developed by Chung and Wee (2011). A production inventory model with random machine breakdown and stochastic repair time was addressed by Widyadana and Wee (2011). Sarkar (2012b) studied an inventory model for finite replenishment rate along with delay in payments where demand and deterioration rate were both time-dependent. Sett et al. (2013) presented a two-warehouse inventory model with quadratically increasing demand and time varying deterioration. An EOQ model for finite production rate and deteriorating items with time dependent increasing demand was established by Sarkar et al. (2013). Most recently, Sarkar (2013) developed a production-inventory model for three different types of continuously distributed deterioration functions.

In this proposed model, an infinite planning horizon inventory model for deteriorating items with power-form inventory dependent demand is developed. Every product has its own maximum life time. After crossing its maximum life time, the product undergoes to deterioration. Based on this idea, we consider time-varying deterioration rate. The rest of the paper is designed as follows: In Section 2, notation and assumptions are provided. In Section 3, the model is formulated with three possible cases. Some numerical examples and sensitivity analysis are presented to illustrate the model in Sections 4 and 5. Finally, conclusions are made in Section 6.

2. Notation and assumptions

The following notation and assumptions are considered to develop the model:

Notation:

- C_h = holding cost per unit ($/unit);
- C_p = purchase cost per unit ($/unit);
- C_d = deterioration cost per unit ($/unit);
- C_0 = ordering cost per order ($/order);
- C_s = selling price per unit ($/unit);
- i_T = ending inventory level of the cycle, i.e., $i_T = i(T)$ (units);
- t_1 = time when the inventory level falls to P_0 (days);
- T = length of the ordering cycle (days);
- $i(t)$ = inventory level at time t (days);
- P = maximum inventory level (unit);
- P_0 = inventory level (unit);
- L = maximum life time of the product (days);
- $Z_1(T)$ = total profit per unit for Case-I ($/unit);
- $Z_2(t_1,T)$ = total profit per unit for Case-II ($/unit);
- $Z_3(T,P)$ = total profit per unit for Case-III ($/unit).

Assumptions:

1. The inventory system deals with a single type of item.
2. The replenishment occurs instantaneously at an infinite rate.
3. The deterioration rate is time dependent as $\theta(t) = \frac{1}{1+L-t}$, where $L > t$ and L is the maximum lifetime of the products at which the total on-hand inventory deteriorates. When t increases, $\theta(t)$ increases and $\mathrm{Lim}_{t\to L}\theta(t)\to 1$ and there is no replacement or repair of deteriorated items during the period under consideration.
4. The inventory-dependent demand rate $D(i(t))$ is taken as the following form:

$$D(i(t)) = \begin{Bmatrix} \alpha(i(t))^{\beta} & \text{if } i(t) > P_0, \\ D & \text{if } 0 \le i(t) \le P_0. \end{Bmatrix}$$

   where $\alpha > 0$, $0 < \beta < 1$, and α and β are known as scale and shape parameters.
5. Shortages are not considered.
6. The lead time is taken as negligible.

3. Mathematical model

We consider an inventory model for inventory-dependent demand with different types of time periods. Depending on this policy, there may arise some cases:

Case (1). If the initial inventory-level is less than or equal to P_0 (i.e., $\mathbf{P} \le \mathbf{P_0}$), then the demand is constant and the inventory model becomes a classical inventory model with constant demand rate (See Fig. 1).

Case (2). If the initial inventory-level is higher than P_0 and P_0 is higher than the ending inventory level i_T (i.e., $\mathbf{P} > \mathbf{P_0} > \mathbf{i_T}$), then the demand rate initially is power-form of one hand inventory, and later becomes constant after the inventory level reaches to P_0 (See Fig. 2).

Case (3). Finally, if the ending inventory level i_T is greater than or equal to P_0 (i.e., $\mathbf{P} > \mathbf{i_T} \ge \mathbf{P_0}$), then the demand rate is power form of one hand inventory (See Fig. 3).

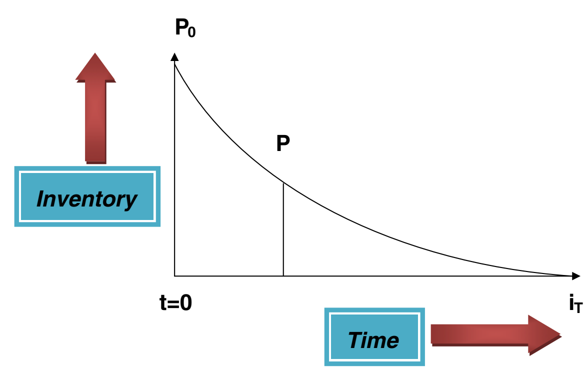

Fig. 1. Inventory versus time: Case 1. $\mathbf{P} \le \mathbf{P_0}$.

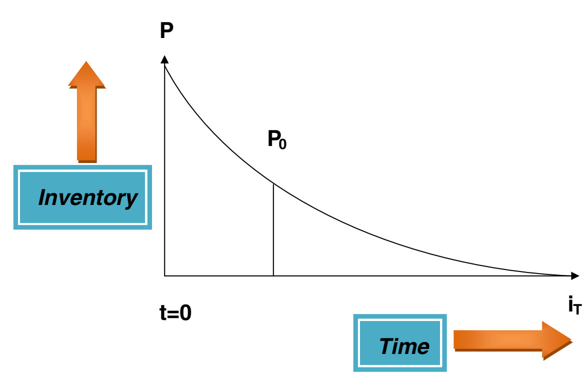

Fig. 2. Inventory versus time: Case 2. $\mathbf{P} > \mathbf{P_0} > \mathbf{i_T}$.