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On complex dynamics of monopoly market

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ABSTRACT

The equilibrium state of a bounded rational monopolist model is studied in this paper. It is assumed that the entire demand function is considered based on some market experiments to produce a quantity which maximizes profit. The stability of equilibrium state of the model is discussed. In addition, some complex dynamical behaviours of the model are illustrated.

1. Introduction

In recent years, many researchers have demonstrated that economic agents may not be fully rational. Even if one tries to do more efforts to perform things correctly, it is important to utilize simple rules previously tested in the past (Naimzada and Ricchiuti, 2008). In economic structure, an economic market may be described as an oligopoly model where few firms manufacture same commodities, or homogeneous commodities. Interdependence is an important characteristic of that market in which each firm must take into account the actions of their competitors in the market so that it can choose the best reaction (Bischi and Lamantia, 2002). Oligopolistic market structures have a distinguishing feature that is the output, pricing and other decisions of one firm affect, and are affected by, the similar decision made by other firms in the market (Fudenberg and Levine, 1998; Gibbons, 1992). The characteristics of oligopoly may be few sellers, either standardized or differentiated products, price interdependence, and relatively difficult entry into and exit from market. Indeed, game theory is one of the important tools in the economists' analytical kit for analyzing the strategic behaviour of this market (Hofbauer and Sigmund, 1998; Webb, 2007). Various empirical works have shown that difference equations have been extensively used to simulate the behaviour of monopolistic markets (Abraham et al., 1997; Elaydi, 2005; Sedaghat, 2003). The present paper extends the work done by Naimzada and Ricchiuti (2008). In Naimzada and Ricchiuti (2008), the author has studied the complex dynamics of monopolistic firms that use a rule of thumb to discover the quantity to produce. It has been used a demand function that does not have an inflection point. Although there are few works in this direction, Naimzada's model gets much simpler since he has achieved a one-dimensional map. In our paper, some difficult cases than the one studied by Naimzada are considered.

The paper is organized as follows: in Section 2, the model is presented. The stability of equilibrium point is discussed in Section 2. Moreover, some simulation and analysis are carried out to show the complex dynamics of the model; from stability and bifurcation to chaos. In the final section, we conclude the obtained results.

2. The model

In this section, a general inverse demand function is considered. It is concave and has the form:

$$p = a - bq^{\alpha}, \alpha \in \mathbb{Z}^{+}. \tag{1}$$

It is well-known that p indicates commodity price, while q is the quantity produced and demanded. In addition, a and b are positive constants. Realistically, q must be positive quantity. It is clear that the downward sloping is guaranteed if: $\frac{dp}{dq} = -\alpha b q^{\alpha-1}$ providing that $b > 0$. Non-negative price is achieved when $q < \sqrt[\alpha]{\frac{a}{b}}$. A simple linear cost function $C(q) = cq$ is presented, where $c \geq 0$ is a marginal cost. It is assumed that the general principle of setting price above marginal cost, $(p-c) > 0$, for each $0 \leq q < \sqrt[\alpha]{\frac{a}{b}}$ and $a > c$. In market, each firm wants to maximize its profit,

$$\pi(q) = pq - C(q) = (a-c)q - bq^{1+\alpha}. \tag{2}$$

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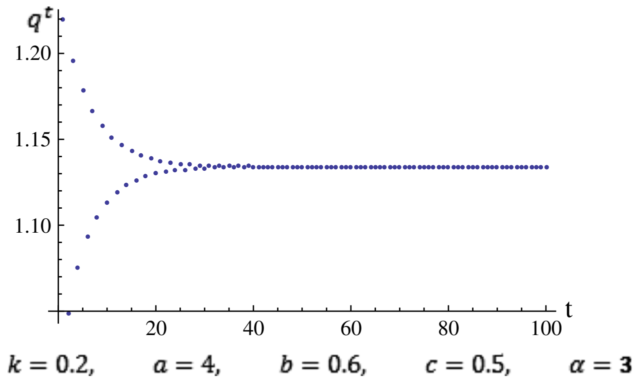

Fig. 1. $k=0.2$, $a=4$, $b=0.6$, $c=0.5$, $\alpha=3$.

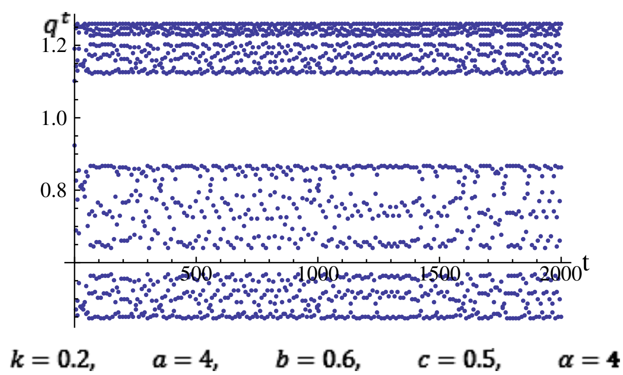

Fig. 2. $k=0.2$, $a=4$, $b=0.6$, $c=0.5$, $\alpha=4$.

This profit is concave and given the following first order condition,

$$\frac{d\pi(q)}{dq}=(a-c)-b(1+\alpha)q^{\alpha}. \tag{3}$$

It has a maximum value if $\overline{q}=\left(\frac{a-c}{b(1+\alpha)}\right)^{\frac{1}{\alpha}}$. It is clear from $\overline{q}$ that positive equilibrium production occurs when $a>c$. Since firm wants to increase its profit, the mechanism used to achieve that is the gradient mechanism. In this mechanism, a positive (negative) variation of profit makes the monopolist to change the quantity produced in the same (opposite) direction from that of the previous period. This mechanism is represented as follows:

$$q^{t+1}=q^{t}+k\frac{d\pi\left(q^{t}\right)}{dq^{t}} \tag{4}$$

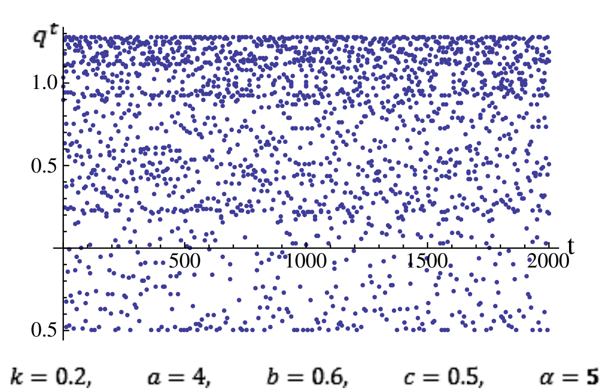

Fig. 3. $k=0.2$, $a=4$, $b=0.6$, $c=0.5$, $\alpha=5$.

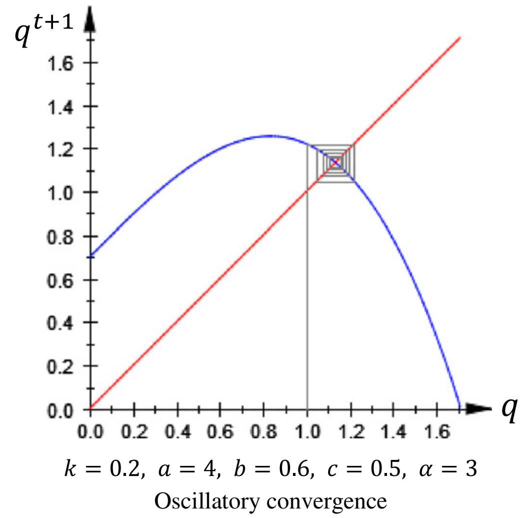

Fig. 4. $k=0.2$, $a=4$, $b=0.6$, $c=0.5$, $\alpha=3$.

where k is the speed of adjustment. Substituting Eq. (3) in Eq. (4), one can get the following one-dimensional nonlinear difference equation:

$$q^{t+1}=q^{t}+k\left((a-c)-b(1+\alpha)q^{\alpha}\right). \tag{5}$$

The dynamical behaviour of map (5) is shown for some different values of α in Figs. 1, 2 and 3. It is shown in Fig. 1 that the nonlinear model goes to a fixed point while the other two figures show it goes to chaotic behaviour.

Proposition. Map (5) has a unique steady state at $\overline{q}$, which is exactly the quantity that maximizes the profit. It is locally stable if: $0<\alpha bk(1+\alpha)\left[\frac{a-c}{b(1+\alpha)}\right]^{\frac{\alpha-1}{\alpha}}<2$. Moreover, there is a period doubling bifurcation if $k=\frac{2}{\alpha b}\left[\frac{b(1+\alpha)}{a-c}\right]^{\frac{\alpha-1}{\alpha}}, \alpha=3,4,5$.

Proof. The map (5) attains its steady state at $q^{t+1}=q^{t}=\overline{q}$. Substituting this condition in Eq. (5), one can easily get the unique steady state $\overline{q}=\left(\frac{a-c}{b(1+\alpha)}\right)^{\frac{1}{\alpha}}$. It is the unique equilibrium state and

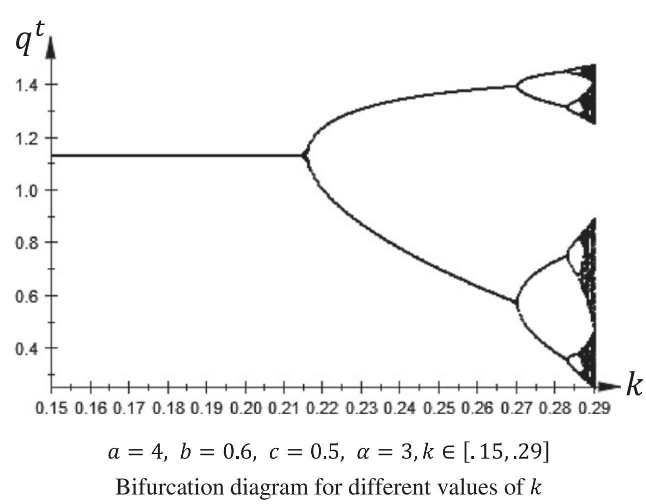

Fig. 5. $a=4$, $b=0.6$, $c=0.5$, $\alpha=3$, $k\in[.15,.29]$.