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Some results on absolute ruin in the perturbed insurance risk model with investment and debit interests

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A B S T R A C T

In this paper, we consider a perturbed compound Poisson risk model with investment and debit interests. Dividends are paid to the shareholders according to a threshold dividend strategy. An alternative assumption is that when the surplus is negative, a debit interest is applied and when the surplus is above a certain positive level, the insurer could earn investment interest. Integro-differential equations with boundary conditions satisfied by the moment-generating function, the nth moment of the present value of all dividends until absolute ruin and the Gerber–Shiu expected discounted penalty function are obtained. Then, we present the explicit expressions for the zero discounted nth moment of the present value of all dividends until absolute ruin in the case of exponential claims. Finally, numerical example is also given to illustrate the effect of the related parameters on the first moment of the present value of all dividends until absolute ruin.

1. Introduction

The classical risk model perturbed by a diffusion was first introduced by Gerber (1970) and has been further studied by many authors during the last few years. See, for example, Tsai and Willmot (2002), Cai and Yang (2005), Wan (2007), Gao and Yin (2008), Wang and Wu (2008), Albrecher and Thonhausera (2008), Gao and Liu (2010), Zhang et al. (2010), Liu and Liu (2011), Zhang (2012) and the references therein. Generally, when the surplus is below zero, we say that ruin occurs in the classical risk theory. But in reality, when the surplus falls below zero or the insurer is on deficit, the insurer could borrow an amount of money equal to the deficits at a debit interest rate to continue his/her business. Meanwhile, the insurer will repay the debts continuously from his/her premium income. Thus, the surplus of the insurer is driven under the debit interest rate when the surplus is negative. The negative surplus may return to a positive level if debts are reasonable. However, it is clear that when the negative surplus reaches some certain level, the surplus is no longer able to recover, and consequently absolute ruin occurs at this moment.

Absolute ruin probability is an important risk measure and has been frequently considered in recent research works. Dassios and Embrechts (1989) derive an explicit expression for the probability of absolute ruin in the case of exponentially distributed individual claim amounts using a martingale approach. Cai (2007) considers the Gerber–Shiu discounted penalty function under absolute ruin. Zhu and Yang (2008) obtain asymptotic results for a more practical case with a higher borrowing rate. Yang et al. (2008) investigate the absolute ruin problems in a multi-layer compound Poisson model with constant interest force. Yuan and Hu (2008) study the absolute ruin in the compound Poisson risk model with nonnegative interest and a constant dividend barrier. Wang et al. (2010) consider the dividend payments in a compound Poisson risk model with credit and debit interests under absolute ruin. Zhang et al. (2011) study an absolute ruin model where claims arrive according to a Markovian arrival process. Li and Liu (2012) investigate a regulated risk process, which is modeled by interest and linear dividend barrier. Under absolute ruin, the expected discounted dividends are derived by PDMP method. Explicit solutions and numerical results are obtained for exponential claims. Bai and Song (2012) consider the probability of random time absolute ruin in the renewal risk model with constant premium rate and constant force of interest. However, there is no work that deals with perturbed compound Poisson risk model with investment and threshold strategy under absolute ruin. This motivates us to investigate such a risk model in this work. Inspired by the work of Gao and Liu (2010), we will extend their results to absolute ruin risk model.

The rest of the paper is organized as follows. In Section 2, we describe the risk model. In Section 3, integro-differential equations with boundary conditions satisfied by the moment-generating function and the nth moment of the present value of all dividends until absolute ruin are derived. In Section 4, we give the integro-differential equations satisfied by the Gerber–Shiu expected discounted penalty function. As applications, in Section 5, we present explicit expressions for the nth moment of the present value of all dividends until absolute ruin for exponential claims when the interest force is zero. Finally, in Section 6, we use numerical

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example to illustrate the impact of the related parameters on the first moment of the present value of all dividends until absolute ruin.

2. The risk model

In the actuarial literature, the surplus process of an insurance company is often modeled by the following perturbed compound Poisson risk process

$$U(t) = u + ct - \sum_{n=1}^{N(t)} X_n + \sigma B(t), \quad t \geq 0, \tag{2.1}$$

where $u \geq 0$ is the initial surplus, $c > 0$ is the gross premium rate, $\{N(t), t \geq 0\}$ is a Poisson process with jump intensity $\lambda > 0$ denoting the number of claims up to time t, and $\{X_n; n \geq 1\}$, representing the sizes of claims and independent of $\{N(t), t \geq 0\}$, is a sequence of independent and identically distributed nonnegative random variables with a common distribution function $F(x)$ which satisfies $F(0) = 0$ and has a positive mean $\mu = \int_0^\infty \overline{F}(x)dx > 0$. Here, $\overline{F}(x) = 1 - F(x)$ is the survival function of $F(x)$. The net profit condition is given by $c > \lambda E[X_i]$. $\sigma > 0$ is a constant, representing the diffusion volatility parameter. $\{B(t), t \geq 0\}$ is a standard Brownian motion with $B(0) = 0$. In addition, $\{N(t), t \geq 0\}$, $\{X_n; n \geq 1\}$ and $\{B(t), t \geq 0\}$ are mutually independent.

Recently, Gao and Liu (2010) consider the compound Poisson risk model perturbed by diffusion with constant interest and a threshold dividend strategy. Under the threshold dividend strategy, whenever the surplus is above b, dividends are paid continuously at a constant rate ε. However when the surplus is below the level b, no dividends are paid. Then, integro-differential equations with certain boundary conditions for the moment-generation function and the nth moment of the present value of all dividends until ruin are derived. Motivated by the work of Gao and Liu (2010), we consider the following extension of model (2.1) which is enriched by investment, debit and threshold strategy.

We denote the aggregate dividends paid in the time interval $[0, t]$ by $D(t)$ and the modified surplus by $U_b(t) = U(t) - D(t)$, which is the insurer's surplus at time t and $U_b(0) = u$. We assume that the insurer pays dividends according to the following strategy governed by threshold parameters $b > 0$ and dividend rate $\varepsilon > 0$. Whenever the modified surplus $U_b(t)$ is below the level b, no dividends are paid and the constant premium income rate is $c_1 > 0$. However, when the modified surplus $U_b(t)$ is above b, dividends are paid continuously at a constant rate ε $(0 < \varepsilon < c_1)$, meanwhile the insurer could earn interest at investment rate $\gamma > 0$. The insurer could borrow an amount of money equal to the deficit at a debit interest force β when the surplus is negative. Meanwhile, the insurer will repay the debts continuously from her/his premium income. But when the surplus is below $-c_1/\beta$, the insurer cannot repay all her/his debts for her/his premium income, then the insurer is no longer allowed to run her/his business. Then the modified surplus process $U_b(t)$ is given by

$$dU_b(t) = \begin{cases} (c_2 + \gamma U_b(t))dt - dS(t) + \sigma dB(t), & U_b(t) > b, \\ c_1 dt - dS(t) + \sigma dB(t), & 0 \leq U_b(t) \leq b, \\ (c_1 + \beta U_b(t))dt - dS(t) + \sigma dB(t), & -c_1/\beta \leq U_b(t) \leq 0, \end{cases} \tag{2.2}$$

where, $c_2 = c_1 - \varepsilon$ is the net premium rate after dividend payments and $S(t) = \sum_{n=1}^{N(t)} X_n$.

We denote the absolute ruin time of the modified surplus process $U_b(t)$ by T_b, which is defined by

$$T_b = \inf\{t \geq 0 : U_b(t) \leq -c_1/\beta\},$$

and $T_b = \infty$ if $U_b(t) > -c_1/\beta$, for all $t \geq 0$. Let $\alpha > 0$ be the force of interest valuation, then the present value of all dividends until T_b is defined by

$$D_{u,b} = \int_0^{T_b} e^{-\alpha t} dD(t). \tag{2.3}$$

An alternative expression for $D_{u,b}$ is

$$D_{u,b} = \varepsilon \int_0^{T_b} e^{-\alpha t} I(U_b(t) > b)dt. \tag{2.4}$$

It is obvious that $0 < D_{u,b} \leq \varepsilon/\alpha$.

In the sequel we will be interested in the moment-generating function

$$M(u, y; b) = E\left[e^{yD_{u,b}}\right], \tag{2.5}$$

and the nth moment function

$$V_n(u; b) = E\left[D_{u,b}^n\right], \quad n \in N, \tag{2.6}$$

with $V_0(u;b) = 1$, and the Gerber–Shiu expected discounted penalty function

$$\Phi(u; b) = E\left[e^{-\alpha T_b} \omega(U_b(T_b-), |U_b(T_b)|) I(T_b < \infty) | U_b(0) = u\right], \tag{2.7}$$

where, $U_b(T_b-)$ is the surplus prior to absolute ruin and $|U_b(T_b)|$ is the deficit at absolute ruin. The penalty function $\omega(x_1, x_2)$ is an arbitrary nonnegative measurable function defined on $(-c_1/\beta, +\infty) \times (c_1/\beta, +\infty)$. Throughout this paper we assume that $M(u, y; b)$, $V_n(u; b)$ and $\Phi(u; b)$ are sufficiently smooth functions in u and y in their respective domains.

3. Integro-differential equations for $M(u, y; b)$ and $V_n(u; b)$

Clearly, the moment-generating function $M(u, y; b)$ behaves differently, depending on whether its initial surplus u is below zero or above the barrier level b. Hence, we write

$$M(u, y; b) = \begin{cases} M_2(u, y; b), & u > b, \\ M_1(u, y; b), & 0 \leq u \leq b, \\ M_3(u, y; b), & -c_1/\beta < u \leq 0. \end{cases} \tag{3.1}$$

For notational convenience, let

$$h_1(u, t) = ue^{\beta t} + c_1\left(e^{\beta t} - 1\right)/\beta, \quad h_2(u, t) = ue^{\gamma t} + c_2\left(e^{\gamma t} - 1\right)/\gamma.$$

Theorem 3.1. *For $0 \leq u \leq b$,*

$$\frac{\sigma^2}{2} \frac{\partial^2 M_1(u, y; b)}{\partial u^2} + c_1 \frac{\partial M_1(u, y; b)}{\partial u} = \lambda M_1(u, y; b) + \alpha y \frac{\partial M_1(u, y; b)}{\partial y} - \lambda\left[\int_0^u M_1(u-x, y; b)dF(x) + \int_u^{u+\frac{c_1}{\beta}} M_3(u-x, y; b)dF(x) + \overline{F}(u + c_1/\beta)\right], \tag{3.2}$$

and, for $u > b$,

$$\frac{\sigma^2}{2} \frac{\partial^2 M_2(u, y; b)}{\partial u^2} + (\gamma u + c_2) \frac{\partial M_2(u, y; b)}{\partial u} = \lambda M_2(u, y; b) + \alpha y \frac{\partial M_2(u, y; b)}{\partial y} - \lambda\left[\int_0^{u-b} M_2(u-x, y; b)dF(x) + \int_{u-b}^u M_1(u-x, y; b)dF(x) + \int_u^{u+\frac{c_1}{\beta}} M_3(u-x, y; b)dF(x) + \overline{F}(u + c_1/\beta)\right], \tag{3.3}$$

and, for $-c_1/\beta \leq u \leq 0$,

$$\frac{\sigma^2}{2} \frac{\partial^2 M_3(u, y; b)}{\partial u^2} + (\beta u + c_1) \frac{\partial M_3(u, y; b)}{\partial u} = \lambda M_3(u, y; b) + \alpha y \frac{\partial M_3(u, y; b)}{\partial y} - \lambda\left[\int_0^{u+\frac{c_1}{\beta}} M_3(u-x, y; b)dF(x) + \overline{F}(u + c_1/\beta)\right], \tag{3.4}$$