Economic Modelling 31 (2013) 659–663

Contents lists available at SciVerse ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/ecmod

Fixed versus variable rate loans under state-dependent preferences

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ARTICLE INFO

Article history:
Accepted 2 January 2013

JEL classification:
D21
D81
G21

Keywords:
Fixed rate loans
State-dependent preferences
Variable rate loans

ABSTRACT

This paper examines the optimal mix of fixed and variable rate loans of a competitive bank facing uncertain funding costs. The bank's preferences are state-dependent in that the utility function depends on a state variable. We show that the optimal amount of loans extended by the bank depends neither on the state-dependent preferences of the bank, nor on the joint distribution of the marginal cost of funds and the state variable. The bank, however, optimally lends less should it be forced to assume all interest rate risk by exclusively extending fixed rate loans. We show further that a non-positive spread between fixed and variable rate loans is no longer a necessary and sufficient condition for the bank to refrain from extending fixed rate loans should the marginal cost of funds be correlated with the state variable in the sense of expectation dependence. State-dependent preferences as such play a pivotal role in determining the bank's optimal choice between fixed and variable rate loans.

1. Introduction

One important aspect of bank asset management is on the choice between fixed and variable rate loans. As pointed out by Santomero (1983), this is a response to the increased variability of deposit rates and the continued deregulation of short-term bank liabilities. Banks use variable rate loans to transfer interest rate risk to their customers for the sake of better risk management.

To study the optimal mix of fixed and variable rate loans, Chang et al. (1995) consider a risk-averse competitive bank that faces uncertain funding costs. The risk-averse behavior of the bank can be motivated by managerial risk aversion (Stulz, 1984), corporate taxes (Smith and Stulz, 1985), costs of financial distress (Smith and Stulz, 1985), and/or capital market imperfections (Froot et al., 1993; Stulz, 1990). Chang et al. (1995) show that the rate differential (spread) between fixed and variable rate loans plays a pivotal role in determining the optimal mix of these two types of loans in general, and that a positive spread between fixed and variable rate loans is both necessary and sufficient to induce the bank to extend fixed rate loans in particular.

The purpose of this paper is to re-examine the results of Chang et al. (1995) under the premise that the competitive bank's preferences are state-dependent.[1] There are legitimate reasons for adopting the state-dependent-preferences approach.[2] As convincingly argued by Briys and Schlesinger (1993), the state-dependent-preferences approach can be used to describe a reduced form of a more complex expected utility model that allows exogenous variations in wealth and relative prices. For example, the state-dependent-preferences approach fits for the scenario that there are exogenous fluctuations in base wealth (Briys et al., 1993), or for the scenario that there is uncertainty about the general price level, but wealth is expressed in nominal terms (Adam-Müller, 2000). The bank as such is assumed to possess a state-dependent utility function that is defined over its profit and a state variable, where the state variable can be interpreted as the business cycle of the economy (Broll and Wong, 2010).[3]

We show that the optimal amount of loans extended by the bank depends neither on the state-dependent preferences of the bank, nor on the joint distribution of the marginal cost of funds and the state variable. The bank, however, optimally lends less should it be forced to assume all interest rate risk by exclusively extending fixed rate loans. Given that the bank's preferences exhibit correlation loving (Eeckhoudt et al., 2007), we show that a non-positive spread between fixed and variable rate loans is sufficient (necessary) but not necessary (sufficient) to induce the bank to refrain from extending fixed rate loans if the marginal cost of funds is positively (negatively) correlated with the state variable in the sense of expectation dependence (Wright, 1987). On the other hand, given that the bank's preferences exhibit correlation aversion (Eeckhoudt et al., 2007), we show that a non-positive spread between fixed and variable rate loans is necessary (sufficient) but not sufficient (necessary) to induce the bank to refrain from extending fixed rate loans if the marginal cost of funds is positively (negatively)

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[1] Karni et al. (1983) provide an axiomatization of expected utility maximizing behavior with subjective probabilities and state-dependent preferences. For a comprehensive introduction of the theory of state-dependent preferences, see Karni (1985).

[2] See Briys and Schlesinger (1993), Broll and Eckwert (1998), Broll and Wong (2002), Lien (2004), and Wong (2012) for other applications of the state-dependent-preferences approach.

[3] See Tsai (2012) and Wong (2011) for an alternative approach using regret aversion to study a bank's optimal interest margin along the line of Wong (1997).

0264-9993/$ – see front matter
http://dx.doi.org/10.1016/j.econmod.2013.01.006

expectation dependent on the state variable. State-dependent preferences as such are crucial for the bank's optimal choice between fixed and variable rate loans.

The rest of this paper is organized as follows. Section 2 delineates our model of a competitive bank facing uncertain funding costs and possessing state-dependent preferences. Section 3 derives the conditions under which variable rate loans dominate fixed rate loans. Section 4 examines the bank's optimal mix of fixed and variable rate loans. The final section concludes.

2. The model

Consider a competitive bank that makes decisions in a single period horizon with two dates, 0 and 1. At date 0, the bank extends two homogeneous classes of fixed and variable rate loans, all of which mature at date 1. The bank finances its loans by variable rate liabilities. The marginal cost of funds, $\tilde{R}_d$, is stochastic and distributed according to a known cumulative distribution function (CDF), $F(R_d)$, over support $[\underline{R}_d, \overline{R}_d]$, where $0<\underline{R}_d<\overline{R}_d$.[4] Throughout the paper, random variables have a tilde (~) while their realizations do not.

We follow Chang et al. (1995) to assume that the one-plus lending rate on variable rate loans, $\tilde{R}_v$, is tied to the random funding cost in that $\tilde{R}_v = \tilde{R}_d + M$, where $M>0$ is an additive mark-up exogenously determined by the competitive market condition.[5] On the other hand, the one-plus lending rate on fixed rate loans, $R_f>0$, is deterministic. The bank is competitive in the sense that its actions influence neither the loan rates nor the marginal cost of funds.

The bank's profit at date 1, $\tilde{\Pi}$, is given by

$$\tilde{\Pi} = R_f L_f + \tilde{R}_v L_v - \tilde{R}_d L - C(L), \quad (1)$$

where $L_f \geq 0$ and $L_v \geq 0$ are the amounts of fixed and variable rate loans, respectively, $L = L_f + L_v$, and $C(L)$ is the cost function of servicing loans. We assume that $C(0) = C'(0) = 0$ and that $C'(L)>0$ and $C''(L)>0$ for all $L>0$. Rearranging terms of Eq. (1) yields

$$\tilde{\Pi} = \left(R_f - \tilde{R}_d\right) L_f + M L_v - C(L), \quad (2)$$

since $L = L_f + L_v$ and $\tilde{R}_v = \tilde{R}_d + M$. Inspection of Eq. (2) reveals that the bank faces no interest rate risk arising from $\tilde{R}_d$ if no fixed rate loans are extended, i.e., $L_f = 0$.

The bank is risk averse and possesses a bivariate state-dependent utility function, $U(\Pi,S)$, defined over its profit at date 1, Π, and the realization of a random variable, $\tilde{S}$, that maps states of nature to real numbers. We denote $G(S)$ as the known CDF of $\tilde{S}$ over support $[\underline{S}, \overline{S}]$, where $\underline{S}<\overline{S}$. To allow the marginal cost of funds, $\tilde{R}_d$, to be correlated with the state variable, $\tilde{S}$, we specify their joint CDF as $H(R_d,S)$ defined over support $[\underline{R}_d, \overline{R}_d] \times [\underline{S}, \overline{S}]$. The state-dependent utility function, $U(\Pi,S)$, exhibits the properties that $U_\Pi(\Pi,S)>0$ and $U_{\Pi\Pi}(\Pi,S)<0$ for all $\Pi \geq 0$ and $S \in [\underline{S}, \overline{S}]$, where the subscripts indicate partial derivatives.

At date 0, the bank chooses the mix of fixed and variable rate loans so as to maximize the expected utility of its profit at date 1:

$$\max_{L_f \geq 0, L_v \geq 0} \mathrm{E}\left[U\left(\tilde{\Pi}, \tilde{S}\right)\right], \quad (3)$$

where $\mathrm{E}(\cdot)$ is the expectation operator with respect to the joint CDF, $H(R_d,S)$, and $\tilde{\Pi}$ is given by Eq. (1). The Kuhn–Tucker conditions for program (3) are given by

$$\mathrm{E}\left\{U_\Pi\left(\tilde{\Pi}^*, \tilde{S}\right)\left[R_f - \tilde{R}_d - C'(L^*)\right]\right\} \leq 0, \quad (4)$$

and

$$\mathrm{E}\left\{U_\Pi\left(\tilde{\Pi}^*, \tilde{S}\right)\left[M - C'(L^*)\right]\right\} \leq 0, \quad (5)$$

where an asterisk (*) signifies an optimal level. If $L_f^*>0$, condition (4) holds with equality. Likewise, if $L_v^*>0$, condition (5) holds with equality. The second-order conditions for program (3) are satisfied given risk aversion and the strict convexity of $C(L)$.

3. Dominance of variable loans over fixed rate loans

Since the bank is risk averse and extending fixed rate loans exposes the bank to interest rate risk arising from $\tilde{R}_d$, it is of interest to examine the conditions under which variable rate loans dominate fixed rate loans, i.e., $L_f^* = 0$ and $L_v^*>0$. In this case, condition (5) holds with equality so that $C'(L_v^*) = M$. Condition (4) can then be written as

$$R_f - \mathrm{E}\left\{\frac{U_\Pi\left[ML_v^* - C(L_v^*),\ \tilde{S}\right]\tilde{R}_d}{\mathrm{E}\left\{U_\Pi\left[ML_v^* - C(L_v^*),\ \tilde{S}\right]\right\}}\right\} - M \leq 0. \quad (6)$$

From the second-order conditions for program (3), we have $L_f^* = 0$ if, and only if, condition (6) holds.

Define the following function:

$$\Phi(R_d) = \int_{\underline{R}_d}^{R_d}\int_{\underline{S}}^{\overline{S}} \frac{U_\Pi[ML_v^* - C(L_v^*), S]}{\mathrm{E}\left\{U_\Pi\left[ML_v^* - C(L_v^*),\ \tilde{S}\right]\right\}}\, \mathrm{d}H(X,S), \quad (7)$$

for all $R_d \in [\underline{R}_d, \overline{R}_d]$. It is evident from Eq. (7) that $\Phi'(R_d)>0$, $\Phi(\underline{R}_d) = 0$, and $\Phi(\overline{R}_d) = 1$. We can as such interpret $\Phi(R_d)$ as a CDF of $\tilde{R}_d$. Hence, condition (6) can be expressed as

$$R_f - \mathrm{E}_\Phi\left(\tilde{R}_d\right) - M \leq 0, \quad (8)$$

where $\mathrm{E}_\Phi(\cdot)$ is the expectation operator with respect to the CDF, $\Phi(R_d)$. Using the covariance operator, $\mathrm{Cov}(\cdot,\cdot)$, with respect to the joint CDF, $H(R_d,S)$, we have[6]

$$\mathrm{E}_\Phi\left(\tilde{R}_d\right) = \mathrm{E}\left(\tilde{R}_d\right) + \frac{\mathrm{Cov}\left\{U_\Pi\left[ML_v^* - C(L_v^*),\ \tilde{S}\right],\ \tilde{R}_d\right\}}{\mathrm{E}\left\{U_\Pi\left[ML_v^* - C(L_v^*),\ \tilde{S}\right]\right\}}. \quad (9)$$

Eq. (9) implies that $\mathrm{E}_\Phi\left(\tilde{R}_d\right)$ is equal to the expected value of $\tilde{R}_d$ plus a risk premium that takes the bank's state-dependent preferences into account.

According to Ingersoll (1987), a random variable, $\tilde{X}$, is said to be conditionally independent of another random variable, $\tilde{Y}$, if $\mathrm{E}\left(\tilde{X}|Y\right) = \mathrm{E}\left(\tilde{X}\right)$ for all realizations of $\tilde{Y}$, where $\mathrm{E}\left(\tilde{X}|Y\right)$ is the expected value of $\tilde{X}$ conditional on $\tilde{Y} = Y$. Ingersoll (1987) proves that $\tilde{X}$ is conditionally independent of $\tilde{Y}$ if, and only if, $\mathrm{Cov}\left[\tilde{X}, \alpha\left(\tilde{Y}\right)\right] = 0$ for all functions, $\alpha(\cdot)$.[7]

[4] An alternative way to model the funding cost uncertainty is to apply the concept of information systems that are conditional cumulative distribution functions over a set of signals imperfectly correlated with $\tilde{R}_d$ (Broll et al., 2012).

[5] Alternatively, we can deviate from Chang et al. (1995) by specifying the one-plus lending rate on variable rate loans as $\tilde{R}_v = (1+m)\tilde{R}_d$, where $m>0$ is a multiplicative mark-up exogenously determined by the competitive market condition. In this case, the variable rate loans no longer provide a perfect hedge against the funding cost uncertainty. We leave this interesting extension for future research.

[6] For any two random variables, $\tilde{X}$ and $\tilde{Y}$, we have $\mathrm{Cov}\left(\tilde{X}, \tilde{Y}\right) = \mathrm{E}\left(\tilde{X}\tilde{Y}\right) - \mathrm{E}\left(\tilde{X}\right)\mathrm{E}\left(\tilde{Y}\right)$.

[7] Ingersoll (1987) shows that two random variables, $\tilde{X}$ and $\tilde{Y}$, are independent if, and only if, $\mathrm{Cov}\left[\alpha\left(\tilde{X}\right), \beta\left(\tilde{Y}\right)\right] = 0$ for all functions, $\alpha(\cdot)$ and $\beta(\cdot)$. Thus, conditional independence contributes to a weaker condition than independence.