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Sales team's initiatives and stock sensitive demand — A production control policy

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ABSTRACT

The present article deals with an optimal production control policy for stock and sales team's initiatives sensitive demand. The capacity of the production quantity and stock of produced items are state variable and the effort of sales-team/agent is a control variable in this model. Finally, a net profit function by trading off procurement cost, cost for effort of sales-team, cost for capacity of production and sales price is maximized by Pontryagin's Maximal Principle. The local and global stability analysis of the dynamical systems is well performed. The proposed model is justified by proper numerical illustration.

1. Introduction

This is common to all enterprises, especially in shopping mall and supermarket, that large piles of consumer goods displayed attract the customers to buy more. The philosophy behind it is that the customers have a scope to choose their better design/quality products among huge stock. Silver and Peterson (1998) observed that sales at the retail level tend to be proportional to the amount of inventory displayed. Ghosh and Chaudhuri (2004) developed an inventory model for a perishable items while the demand of end customers is dependent on two level of stocks. Numerous works in stock-dependent demand pattern have attracted the researchers as well as practitioners, in past decades. The research articles related to the stock-dependent demand pattern of Urban (1992), Chang (2004), Hou and Lin (2006), Jolai et al. (2006), Soni and Shah (2008), Goyal and Chang (2009), Min and Zhou (2009), Min et al. (2010) and Sarkar (2012a) are worth mentioning. The inventory/production model on stock-dependent demand for static system has a large volume whereas the models on stock-dependent demand in dynamical system are few in number. Recently, Sana (2011a,b, 2012) investigated the inventory models in a dynamical system when the demand rates of two homogeneous products are dependent on stock-level, efforts of sales-team and variable sales prices. In these models, the replenishment rates depend on the level of stock and storage capacities and the effort of sales team and sales prices of the products are considered as control variables of the system.

Promotional effort is a common and effective strategy which provokes or motivates the customers to buy more. Generally, promotional strategies include delay-in-payments, free gifts, discounts, displays, special services and advertising, among others. In oligopoly marketing system, the sales team has a lot of pressure to boost sale their goods. The sales team attracts the customers by their promotional efforts which are called initiatives by sales team in practice. Goyal and Gunasekaran (1995) formulated an integrated production–inventory–marketing model introducing the influences of different marketing policies such as the price per unit product and the advertisement frequency on the demand of a deteriorating item. Nair and Tarasewich (2003) pointed out the optimal design of promotional efforts including free gifts, discounts, special services, etc. Krishnan et al. (2004) maximized the revenue, applying the promotional strategies including price costs, displays, free goods and advertising. Sana and Chaudhuri (2008) developed an inventory model for stock and advertising sensitive demand. Szmerekovsky and Zhang (2009) investigated the optimal pricing strategy and two-tier advertising level between one manufacturer and one retailer for retail price and advertising sensitive demand of the end customers. He et al. (2009) discussed the supply chain contracts and coordination while the retailer faced both effort and price dependent stochastic demand. Xie and Wei (2009) and Xie and Neyret (2009) analyzed the optimal collaborative advertising strategies and equilibrium pricing in two-layer supply chain. Sana (2010a) discussed a multi-item EOQ (Economic Order Quantity) model for deteriorating and ameliorating items when the time varying demand is influenced by enterprises' initiatives like advertising media and salesmen's initiatives.

The conventional production scheduling approach, known as static scheduling (Cárdenas-Barrón, 2007; Cárdenas-Barrón et al., 2011, 2012a,b,c; Sarkar, 2012b; Sarkar and Sarkar, 2013), provides usually suboptimal schedule. In practice, this schedule turn out to be impractical because of its unrealistic assumptions. Generally speaking, the real manufacturing systems are complex and dynamic with a big number of products and processes, with many production levels, and subject to random disturbances. Consequently, the production and capacity of production quantity are agile, adaptive, and dynamic in nature. Benhadid et al.(2008) presented an optimal control problem where inventory level and rate of manufacturing are considered as state variable and control variable respectively. A few authors enlightened on optimal control theory for production planning and control. In this field, the works of Axsater (1985), Hedjar et al. (2004),

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Khemlnitsky and Gerchak (2002), Riddalls and Bennett (2001), Zhang et al. (2001), Sana (2010b) should be mentioned.

This article focuses especially the application of optimal control theory to the production planning problem while the capacity of the manufacturing process is dynamic in nature. In this model, the demand of the products is varying with the effort of sales team/sales agent and stock of the goods. The capacity of resources for production is not unlimited, in practice. Consequently, the capacity of production quantity has an upper bound. The production and capacity of production quantity are state variables and the effort of sales team/agent is considered as control variable. Finally, the net profit of the manufacturer is maximized by Pontryagin's Maximal Principle. The rest of the paper is organized as follows: Section 2 provides the notation of the parameters and variables of the model. Formulation of the model is described in Section 3. Boundedness of the dynamical system is proved in Section 3.1. Local stability and global stability analysis have been done in Sections 3.2 and 3.3 respectively. Optimal goal of the policy is obtained in Section 3.4. To test the proposed model, numerical example is given in Section 4. Section 5 concludes the proposed model.

2. Notation

The following notations are used to develop the proposed model.

$C_{\max}$	Maximum possible capacity of the production.
$C(t)$	On-hand capacity for production.
$Q(t)$	On-hand stock of products.
$\dot{Q}$	Time derivative of Q.
$\dot{C}$	Time derivative of C.
$N(t)$	Volume of initiatives of sales team/agents, a control variable.
$P(t)$	Production rate at time t.
$D(t)$	Demand rate of finished product at time t.
$\tilde{Q}$	Initial stock, i.e., $\tilde{Q}=Q(0)$.
$\tilde{C}$	Initial capacity, i.e., $\tilde{C}=C(0)$.
$\tilde{N}$	Initial volume of initiatives of sales team/agents.
r	Proportionate rate of on-hand stock and available capacity, a potential growth rate of production.
τ	A positive constant which is estimated from a previous knowledge of demand, when demand is dependent on initiatives of sales team/agents and stock of goods.
q	Growth rate of capacity **C** driven by growth of production resources.
γ	Loss rate of **C** due to alteration of resources caused by aggressive demand of the agents.
w	Selling price per unit product.
w_0	Procurement cost per unit product.
w_1	Cost per unit effort of sales team/agent.
w_2	Cost per unit capacity.
δ	Difference of rate of interest and inflation of currency, i.e., $\delta=$(rate of interest-inflation of currency).
R	Discounted present value of revenue earned at time t.
J	Net profit of the system.

3. Formulation of the model

Let (Q, C, N) be on hand stock, capacity of production and volume of initiatives by sales team respectively. Now, the production rate is controlled by on-hand stock and production capacity of goods as follows:

$$P(t)=rQ\left(1-\frac{Q}{C}\right) \tag{1}$$

where $r(>0)$ is potential growth rate of production. The production rate, change of production quantity, is proportional to the on hand stock and capacity of the production system. Here, $\lim_{Q\to C}P(t)\to 0$ that is quite natural.

The demand rate of the finished products is

$$D(t)=\tau\left(1-\frac{1}{1+N}\right)Q \tag{2}$$

where $\tau(>0)$ is a positive constant which is estimated by previous knowledge of data. Here, $\frac{\partial D}{\partial N}=\frac{\tau Q}{(1+N)^2}>0\forall Q\in(0,\infty)$ and $\lim_{N\to\infty}D(t)\to\tau Q(t)$, i.e. $D(t)\in[0,\tau Q(t)]$. This mathematical formulas agree with the real situation, because infinite volume of sales team/agent does not generate infinite demand although the demand rate D(t) is an increasing function of N. Therefore, the governing differential equation for on-hand stock is

$$\dot{Q}=P(t)-D(t)=rQ\left(1-\frac{Q}{C}\right)-\tau\left(1-\frac{1}{1+N}\right)Q=F(Q,C) \tag{3}$$

Now, the capacity of production (C) follows the differential equation:

$$\dot{C}=qC\left(1-\frac{C}{C_{\max}}\right)-\gamma NC=G(C) \tag{4}$$

where $q(>0)$ is growth rate of capacity C driven by growth of production resources and $\gamma(>0)$ is loss rate of C due to alteration of resources caused by aggressive demand of the agents. $C_{\max}$ is the maximum capacity of the production though the production system is agile, adaptive, and dynamic. Therefore, the differential equations of the dynamical system is

$$\left\{\begin{array}{c}\dot{Q}=rQ\left(1-\frac{Q}{C}\right)-\tau\left(1-\frac{1}{1+N}\right)Q=F(Q,C)\\ \dot{C}=qC\left(1-\frac{C}{C_{\max}}\right)-\gamma NC=G(C)\\ with\ \ Q(0)=\tilde{Q}\ and\ C(0)=\tilde{C}\end{array}\right\} \tag{5}$$

3.1. Boundedness of the system

Lemma 1. All solutions of Eq. (5) which start in R_2^+ are uniformly bounded.

Proof. Let the function

$$U(Q,C)=Q+\frac{1}{L}C \tag{6}$$

where L is a positive constant. The time derivative of Eq. (6) is

$$\dot{U}(Q,C)=\dot{Q}+\frac{1}{L}\dot{C}=rQ\left(1-\frac{Q}{C}\right)-\frac{\tau NQ}{1+N}+\frac{1}{L}\left[qC\left(1-\frac{C}{C_{\max}}\right)-\gamma NC\right] \tag{7}$$

For each $s>0$, we have

$$\begin{aligned}\dot{U}+sU\ &=rQ\left(1-\frac{Q}{C}\right)-\frac{\tau NQ}{1+N}+\frac{1}{L}\left[qC\left(1-\frac{C}{C_{\max}}\right)-\gamma NC\right]\\ &+s\left(Q+\frac{1}{L}C\right)<rQ-\frac{\tau NQ}{1+N}+sQ\\ &+\frac{1}{L}(qC+sC-\gamma NC)<\left|r-\frac{\tau N}{1+N}+s\right|Q\\ &+\frac{1}{L}|q+s-\gamma N|C<\left[\left|r-\frac{\tau N}{1+N}+s\right|+\frac{1}{L}|q+s-\gamma N|\right]\\ &C_{\max}\text{ as }0\le Q\le C\le C_{\max}.\end{aligned}$$

Therefore, $\dot{U}+sU<k$ where $k=\left[\left|r-\frac{\tau N}{1+N}+s\right|+\frac{1}{L}|q+s-\gamma N|\right]C_{\max}$.