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Model reference adaptive expectations in Markov-switching economies

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ABSTRACT

This paper offers a theory of model reference adaptive beliefs as a selection device in Markov-switching economies under equilibrium indeterminacy. Consistent with the classical rational choice paradigm, our theory requires that endogenous expectations be replaced with a general-measurable function of the observable states of the model, to be determined optimally. This forecasting function is derived as the regime-independent feedback control minimizing the mean-square deviation of the equilibrium path from the corresponding perfect-foresight state motion (the reference model). We show that model reference adaptive expectations always generate a rational expectations equilibrium, irrespective of the presence of nonlinearities and/or imperfect information. Under equilibrium indeterminacy, this forecasting mechanism enforces the unique mean-square stable solution producing nearly perfect-foresight dynamics.

1. Introduction

Since the early work of Muth (1961) and Lucas (1972), rational expectations (RE) have been widely adopted as the benchmark model of expectation formation in macroeconomics. It essentially reduces to the assumption that the prediction made by the forecaster, conditional on all the information available at the time the prediction is made, be consistent with the forecast model derived from the underlying economic structure.

The RE hypothesis has been severely criticized on the ground of its implausibly strong implications. In particular, it is silent about the process by which economic agents translate current information into optimal forecasts, which are simply assumed not systematically different from equilibrium outcomes (Lucas and Prescott, 1971). In this respect, the learning approach has provided a natural interpretation of RE as asymptotic outcomes of a well-specified learning process, in which boundedly rational agents engage to form estimates of the true (unknown) economy they act in (e.g. Evans and Honkapohja, 2001; Marcet and Sargent, 1989). According to this approach, the agents' subjective beliefs are derived as forecasts from an estimation model, which combines a parametric description of the model dynamics (the perceived law of motion) and an estimation algorithm (least squares or Bayesian updating).

The aim of this paper is to present an optimization-based theory of expectations formation as a selection device in multiple equilibrium model economies. According to this theory, forward-looking agents formulate (and coordinate upon) subjective beliefs on the basis of a well-specified dynamic criterion, which goes beyond the static minimum mean-square error one – the one at the core of the RE paradigm. This approach, which traces back to Basar (1989), complies with a broader definition of rationally formed expectations, according to which agents are endowed with a general-measurable forecasting function, which is to be chosen optimally given the available information (the measurement) on the state(s) of the underlying model economy.

More specifically, we think of unobservable subjective expectations as the aggregate decision of the economic agents as to their best estimate of some future economic state(s), to be taken on the basis of a *model reference adaptive* criterion. Subjective expectations are then derived from an adaptive forecasting model in which the actual model dynamics are forced to track the evolution the system would have if the agents were able to form an exact prediction of the future realizations of the endogenous variables. Under imperfect state observability, the agents are not able to form subjective forecasts on the basis of perceived laws of motion that are consistent with the RE solution(s). In this view, imperfectly informed agents are thought of as adaptively revising their (best) estimate of the future variables governing the dynamics of the economic system as new observations are generated.

We describe our approach in the context of forward-looking stochastic systems which introduce regime-switching and imperfect information into the standard linear model of Evans and Honkapohja (2001).[1] As a main contribution, we show that model reference adaptive expectations always generate a rational expectations equilibrium, irrespective of the presence of nonlinearities and/or imperfect information. When the model equilibrium is indeterminate, this rational forecasting mechanism is able to pin down the unique mean-square stable solution producing

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[1] For the linear case, the reader is referred to Carravetta and Sorge (2010).

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nearly perfect-foresight dynamics. In fact, the adaptive forecasting (belief) function is able to recover all the conditional (rational) expectations terms entering the canonical MSRE systems, and thus selects a particular (history-dependent) RE equilibrium in the solution set. As emphasized by Farmer (2002), multiple equilibrium models create room for adaptive rules, or *belief functions*, to select a particular equilibrium. Specifications of beliefs are restricted by the requirement that, in a stationary environment, the forecasting function should not generate systematic forecast errors. While Farmer (2002) puts forward a quite ad hoc mechanism for the formation of expectations, the model reference adaptive rule presented in this paper is the outcome of a well-defined dynamic optimization/estimation problem, which always fulfills the rationality requirement. Moreover, and differently from Farmer (2002), it does not require that the agent possess a priori knowledge of the structure of the model solution.

The paper is organized as follows. Section 2 briefly reviews the related literature. In Section 3 we present the basic framework of analysis, and describe our approach to expectations formation. In Section 4, we study the model dynamics under model reference adaptive expectations, and also discuss the issue of equilibrium stability. While the analysis here is for an abstract macroeconomic system, an illustrative application to the New Keynesian framework is provided in Section 5. Section 6 concludes.

2. Related literature

Our analysis clearly relates to several lines of research on the process of expectations formation in forward-looking models. The well-known learning approach in macroeconomics focuses on the way systematic forecasting biases are eliminated over time (e.g. Evans and Honkapohja, 2001; Marcet and Sargent, 1989). Specifically, the adaptive learning literature endows boundedly rational agents with a forecasting model – the perceived law of motion of the economy – which can be an arbitrary function of past endogenous and past and current exogenous variables, and has to be optimally parameterized based on new data and observable (past) forecast errors. RE equilibria are thus regarded as asymptotic outcomes of this learning process, whenever conditions for convergence of agents' beliefs to the equilibrium values hold. Our analysis differs from the learning literature in two crucial respects. First, it posits that forward-looking agents form their beliefs via a dynamic optimization criterion within a correctly specified (parametric) forecasting rule; however, under unobserved state variables, the agents can no longer be thought of as employing a forecasting model which is consistent in form with any of the RE solutions. Second, we are not concerned with asymptotic convergence properties, as subjective forecasts formed according to our adaptive approach always coincide – as functions of the common measurable space – with RE, even in finite horizon economies. Though methodologically related, our method also differs from the Bayesian learning literature (e.g. Bullard and Suda, 2008; McGough, 2003), as these studies typically assume that agents employ filtering techniques to update estimates of (possibly time-varying) parameters within not fully rational forecasting functions. Rather, our approach posits that forward-looking agents update their (best) estimate of the (hidden) variables governing the model dynamics as new data is released, when only a limited information set – the measurement process – is available to them.

From a computational perspective, our work is also related to the recent and increasing literature on Markov-switching rational expectations (MSRE) models, that is stochastic difference systems in which the parameters governing the dynamic behavior of the equations are functions of a discrete-state Markov chain. Since able to account for parameter instability and yield quantitatively different responses of macroeconomic variables to fundamental shocks from those implied by fixed regime models, MSRE systems have recently been advocated to investigate the role of regime-switching monetary policy in New Keynesian frameworks (e.g. Davig and Leeper, 2007) or rather to gauge the effects of uncertainty over structural parameters governing the optimal behavior of rational agents (e.g. Liu et al., 2009).

From a technical viewpoint, regime dependency engenders structural nonlinearities which preclude applicability of standard solution tools for linear RE systems, such as Blanchard and Kahn (1980)'s, King and Watson (2002)'s and Sims (2002)'s. In this respect, a number of authors have been interested in deriving determinacy (local uniqueness) conditions for RE equilibria to MSRE models. In their seminal contribution to the generalization of the Taylor principle, Davig and Leeper (2007) study how regime-switching alters the determinacy properties of RE solutions and provide analytical restrictions on monetary policy behavior to ensure (local) uniqueness of the equilibrium path. The nonlinearity problem is addressed by introducing a two-step solution method that consists of studying an augmented system which is linear in fictitious variables, the latter coinciding with the actual ones in some of the regimes, and then using the solution to the linear representation in order to construct solutions for the original nonlinear system.

From a more general perspective, Farmer et al. (2009, 2011) have provided a series of characterization results for the set of minimal state variable (MSV) solutions as well as the full set of RE equilibria – also sunspot ones – to MSRE frameworks, which satisfy a suitable stability concept. Their approach rests on expanding the state-space of the underlying stochastic system and to focus on an equivalent model in the expanded space that features state-invariant parameters. Furthermore, Farmer et al. (2009) demonstrate an equivalence property between determinacy for MSRE models and mean-square stability in a class of Markov jump autoregressive systems.

Our approach to the analysis of regime-switching models differs from the mentioned studies in several respects. First, unlike other approaches which posit the presence of RE, we rather focus on a different model for expectations formation. The latter is explicitly designed to handle dynamic models with an imperfect information structure. In many macroeconomic environments, in fact, variables of interest are typically observed with some delay, and only lagged values of them can be exploited for the formation of expectations (e.g. Collard and Dellas, 2004; Mankiw and Reis, 2002). Also, observability of some relevant economic variables, like factor productivity, can be plagued by measurement errors. From a different perspective, economic policy is typically conducted under substantial uncertainty about the state of the economy and the timing of structural disturbances (e.g. Svensson and Woodford, 2003). Intuitively, a fundamental problem of inference arises, as agents must gather information on unobserved variables from observed ones.[2] Under regime-switching, the possibility of future changes in the model's structure is also crucial to the determination of agents' expectations. In fact, the problem of learning about future (uncertain) Markov regimes cannot be disentangled from that of filtering unobservable (current and expected) variables. As a consequence, methods generally employed to handle learning in models with regime-switching exogenous drifts (e.g. Schorfheide, 2005) cannot be adapted to study MSRE models with imperfect information. The present paper offers some new insights on these issues for a given class of regime-switching systems, namely those involving lagged expectations.[3]

[2] Pearlman et al. (1986) were the first to address the partial information issue in RE models in the fixed regime setting. While generalized to regime-switching frameworks, our analysis also departs from Pearlman et al. (1986) in that it develops an adaptive control technique which jointly exploits Kalman filtering and stochastic control theory. Furthermore, we allow the observation process to be regime-dependent. See also, Lungu et al. (2008), Shibayama (2011) for a different approach to solution of linear RE systems with imperfect information.

[3] Lagged expectations can be associated with different microfoundations, like the presence of staggered-price setting under past information (e.g. Woodford, 2003), information stickiness (e.g. Mankiw and Reis, 2002) or imperfect information in monetary policy-making (e.g. McCallum and Nelson, 2000).