



Existence of singularity bifurcation in an Euler-equations model of the United States economy: Grandmont was right

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ABSTRACT

Grandmont (1985) found that the parameter space of the most classical dynamic general-equilibrium macroeconomic models are stratified into an infinite number of subsets supporting an infinite number of different kinds of dynamics, from monotonic stability at one extreme to chaos at the other extreme, and with all forms of multiperiodic dynamics between.

But Grandmont provided his result with a model in which all policies are Ricardian equivalent, no frictions exist, employment is always full, competition is perfect, and all solutions are Pareto optimal. Hence he was not able to reach conclusions about the policy relevance of his dramatic discovery. As a result, Barnett and He (1999, 2001, 2002) investigated a Keynesian structural model, and found results supporting Grandmont's conclusions within the parameter space of the Bergstrom–Wymer continuous-time dynamic macroeconomic model of the UK economy. That prototypical Keynesian model was produced from a system of second order differential equations. The model contains frictions through adjustment lags, displays reasonable dynamics fitting the UK economy's data, and is clearly policy relevant. In addition, results by Barnett and Duzhak (2010) demonstrate the existence of Hopf and flip (period doubling) bifurcation within the parameter space of recent New Keynesian models.

Lucas-critique criticism of Keynesian structural models has motivated development of Euler equations models having policy-invariant deep parameters, which are invariant to policy rule changes. Hence, we continue the investigation of policy-relevant bifurcation by searching the parameter space of the best known of the Euler equations general-equilibrium macroeconomic models: the path-breaking Leeper and Sims (1994) model. We find the existence of singularity bifurcation boundaries within the parameter space. Although never before found in an economic model, singularity bifurcation may be a common property of Euler equations models, which often do not have closed form solutions. Our results further confirm Grandmont's views.

Beginning with Grandmont's findings with a classical model, we continue to follow the path from the Bergstrom–Wymer policy-relevant Keynesian model, to New Keynesian models, and now to Euler equations macroeconomic models having deep parameters.

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1. Introduction

1.1. The history

Grandmont (1985,1998) found that the parameter space of even the simplest, classical general-equilibrium macroeconomic models are stratified into bifurcation regions. This result changed the prior common view that different kinds of economic dynamics can only be produced by different kinds of structures. But he provided that result with a model in which all policies are Ricardian equivalent, no frictions exist, employment is always full, competition is perfect, and

all solutions are Pareto optimal. Hence he was not able to reach conclusions about the policy relevance of his dramatic discovery. Years of controversy followed, as evidenced by papers appearing in Barnett et al. (2004) and Barnett et al. (2005). The econometric implications of Grandmont's findings are particularly important, if bifurcation boundaries cross the confidence regions surrounding parameter estimates in policy-relevant models. Stratification of a confidence region into bifurcated subsets seriously damages robustness of dynamical inferences.²

The dramatic transformation of views precipitated by Grandmont's paper was criticized for lack of policy relevance. As a result,

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² We assume that parameters are fixed and focus on the implications of bifurcation for robustness of inferences. But if parameters can move over time, as in Swamy et al. (2005), the implications of bifurcation are even more serious.

Barnett and He (1999, 2001, 2002) investigated a continuous-time traditional Keynesian structural model and found results supporting Grandmont's conclusions. Barnett and He found transcritical, codimension-two, and Hopf bifurcation boundaries within the parameter space of the Bergstrom–Wymer continuous-time dynamic macroeconomic model of the UK economy. That highly regarded Keynesian model was produced from a system of second order differential equations. The model contains frictions through adjustment lags, displays reasonable dynamics fitting the UK economy's data, and is clearly policy relevant. See Bergstrom and Wymer (1976), Bergstrom (1996), Bergstrom et al. (1994), Bergstrom et al. (1992), and Bergstrom and Nowman (2006). Barnett and He found that bifurcation boundaries cross confidence regions of parameter estimates in that model, such that both stability and instability are possible within the confidence regions.

Barnett and Duzhak (2008,2009) have explored bifurcation within the more recent class of New Keynesian models. Those two papers included forward-looking and current-looking models, as well as hybrid models having both forward and current-looking features. They found Hopf and flip (period doubling) bifurcation, with the setting of the policy parameters influencing the existence and location of the bifurcation boundary. No other forms of bifurcation were found within the three-equations log-linearized New Keynesian models. One surprising result is the finding that a common setting of a parameter in the future-looking New-Keynesian model can put the model directly onto a Hopf bifurcation boundary.

The Lucas critique has motivated development of Euler-equations general-equilibrium macroeconomic models. Hence, we continue the investigation of policy relevant bifurcation by searching the parameter space of the best known of the policy relevant Euler-equations macroeconomic models: the path-breaking Leeper and Sims (1994) model. The results further confirm Grandmont's views, but with the finding of an unexpected form of bifurcation: singularity bifurcation. Although known in engineering and mathematics, singularity bifurcation has not previously been encountered in economics. Barnett and He (2004, 2006) have made clear the mathematical nature of singularity bifurcation and why it is likely to be common in the class of modern Euler equation models rendered important by the Lucas critique.

Leeper and Sims' model consists of differential equations with a set of algebraic constraints. Our analysis reveals the existence of a singularity bifurcation boundary within a small neighborhood of the estimated parameter values. When the parameter values approach the singularity boundary, one eigenvalue of the linearized part of the model moves rapidly to infinity, while other eigenvalues remain bounded. On the singularity boundary, the number of differential equations will decrease, while the number of algebraic constraints will increase. Such change in the order of dynamics has not previously been found with macroeconomic models. But we find from the relevant theory that singularity bifurcation may be a common property of Euler equations models. The dramatic implications of singularity bifurcation are not limited to the change in the dimension of the dynamics on the bifurcation boundary. The nature of the dynamics on one side of a singularity bifurcation boundary is very different from the nature of the dynamics on the other side, although of the dimension of the dynamics is the same on both sides. Knowledge of the location of a bifurcation boundary is very important, even if there is no chance that the economy will drop into the lower dimensional dynamics directly on that boundary.

Beginning with Grandmont's findings with a classical model, we continue to follow the path from the Bergstrom–Wymer policy-relevant Keynesian model, to New Keynesian macroeconomic models, and now to Euler equations models having deep parameters. At this stage of our research, we believe that Grandmont's conclusions appear to hold for all categories of dynamic macroeconomic models, from the oldest to the newest.

1.2. The Leeper and Sims model

Various relevant dynamic macroeconomic models have been established in the literature.³ Of particular importance is the Leeper and Sims (1994) Euler equations stochastic-dynamic general-equilibrium model intended to address such issues as the Lucas critique (Lucas (1976)) for the US economy. Similar models are developed in Kim (2000) and others, but the Leeper and Sims model was the seminal model in that literature.

The dimension of the state space in the Leeper and Sims model is substantially lower than in the Bergstrom, Norman, and Wymer UK model. However, the dimension is still too high for complete analysis by generally available analytical approaches. By numerical methods complementing theoretical analysis, we find that the dynamics of the Leeper and Sims model is complicated by its structure as an Euler equations model, since such models usually have no closed form algebraic solution.

In this paper, we are interested in how the dynamic behavior of the model is affected by its parameter settings. We find that the order of the dynamics of the Leeper and Sims model can change within a small neighborhood of the estimated parameter values. As parameters change within that neighborhood, one eigenvalue of the linearized part of the model can move quickly from finite to infinite and back again to finite. A large stable eigenvalue characterizes the case in which some variables can respond rapidly to changes of other variables, while a large unstable eigenvalue corresponds to the case in which rapid diversion occurs of one variable from other variables. Infinity eigenvalue implies existence of pure algebraic relationships among the variables. This sensitivity to the setting of the parameters presents serious challenges to the robustness of dynamical inferences. The source of the problem is the nature of the mapping from the Euclidean parameter space to the function space of dynamical solutions.

Change in the order of the dynamic part of the system in response to small changes in parameter settings is a fundamental property of the Leeper and Sims model and corresponds to a class of bifurcations known to engineers and mathematicians as “singularity” bifurcations. To our knowledge, this is the first discovery of singularity bifurcation in macroeconomic models; but appears to be closely connected with the structure of Euler equations models.

2. The model

The Leeper and Sims (1994) Euler-equations, stochastic, general-equilibrium model includes the dynamic behavior of consumers, firms, and government. With the parameters of consumer and firm behavior being the deep parameters of tastes and technology, those parameters are invariant to government policy rule changes.⁴ These models contain dynamic subsystems consisting of ordinary

³ Among those models that have direct relevance to this research are the high-dimensional continuous-time macroeconomic models of Bergstrom and Wymer (1976), Bergstrom and Nowman (2006), Grandmont (1998), Leeper and Sims (1994), Powell and Murphy (1997) and Kim (2000). Surveys of relevant macroeconomic models are available in Bergstrom (1996) and in several textbooks such as Gandolfo (1996) and Medio (1992). General theory of economic dynamics is provided, in Boldrin and Woodford (1990) and Gandolfo (1996). Various bifurcation phenomena are reported in Bala (1997), Benhabib (1979), Medio (1992), Gandolfo (1996), and Nishimura and Takahashi (1992). Focused studies of stability are conducted in Grandmont (1998), Scarf (1960), and Nieuwenhuis and Schoonbeek (1997). Barnett and Chen (1988) empirically found chaotic dynamics in economics. Bergstrom et al. (1994) investigate stabilization of macroeconomic models using policy control. Wymer (1997) describes several mathematical frameworks for the study of the structural properties of macroeconomic models.

⁴ Several similar models have been developed in Kim (2000) and in Binder and Pesaran (1999).

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