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Population dynamics and utilitarian criteria in the Lucas–Uzawa Model

Simone Marsiglio [a,c,*], Davide La Torre [b]

[a] Department of Public Policy and Public Choice, University of Eastern Piedmont Amedeo Avogadro, via Cavour 84, I-15121, Alessandria, Italy
[b] University of Milan, Department of Economics, Business and Statistics, via Conservatorio, 7, I-20122 Milan, Italy
[c] School of Economics, University of East Anglia, NR4 7TJ, Norwich, UK

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ABSTRACT

This paper introduces population growth in the Uzawa–Lucas model, analyzing the implications of the choice of the welfare criterion on the model's outcome. Traditional growth theory assumes population growth to be exponential, but this is not a realistic assumption (see Brida and Accinelli, 2007). We model exogenous population change by a generic function of population size. We show that a unique non-trivial equilibrium exists and the economy converges towards it along a saddle path, independently of population dynamics. What is affected by the type of population dynamics is the dimension of the stable manifold, which can be one or two, and when the equilibrium is reached, which can happen in finite time or asymptotically. Moreover, we show that the choice of the utilitarian criterion will be irrelevant on the equilibrium of the model, if the steady state growth rate of population is null, as in the case of logistic population growth. Then, we show that a closed-form solution for the transitional dynamics of the economy (both in the case population dynamics is deterministic and stochastic) can be found for a certain parameter restriction.

1. Introduction

In standard economic growth theory, population is assumed to grow at an exogenous and exponential rate. This assumption has been firstly introduced by the Solow–Swan model (1956) and it has been applied also to following models with optimizing behavior, as the single-sector Ramsey–Cass–Koopmans (1965) model and the two-sector Lucas–Uzawa (1988) model. Such an assumption however is not without consequences for the analysis of growing economies. In fact, exponential population growth models imply unconstrained growth of population size. However, most populations are constrained by limitations on resources, at least in the short run, and none is unconstrained forever. For this reason, firstly Malthus (1798) discusses about the inevitable dire consequences of exponential growth of the human population of the earth. Recently, Brida and Accinelli clearly state: *"The simple exponential growth model can provide an adequate approximation to such growth only for the initial period because, growing exponentially, as $t \to \infty$, labor force will approach infinity, which is clearly unrealistic. As labor force becomes large enough, crowding, food shortage and environmental effects come into play, so that population growth is naturally bounded. This limit for the population size is usually called the carrying capacity of the environment".*

Some decades ago, Maynard Smith (1974) concluded that the growth of natural populations is more accurately depicted by a logistic law. This result has been recently used to claim that such a dynamics can probably better describe also human population growth. In fact, several studies support the idea that human population growth is decreasing and tending towards zero[1] (as Day, 1996). Even the Belgian mathematician Verhulst in the XIX century studies this idea; using data from the first five U.S. censuses, he makes a prediction in 1840 of the U.S. population in 1940 and was off by less than 1%. Moreover, based on the same idea, he predicts the upper limit of Belgian population; more than a century later, but for the effect of immigration, his prediction looks good (Verhulst, 1838). More recently, several studies try to understand which function fits better human population dynamics, showing that the exponential growth is reductive. For example, population dynamics can be described through a non-autonomous differential equation as $N_t = N_t g((N_t)$, where $g(N) = \sum_{i=0}^{m} g_i N^i$. The estimation of the parameters g_i can be done by using, for instance, fractal-based methods and penalization methods[2] as proposed and well-illustrated in Kunze et al. (2007a, 2007b, 2009a, 2009b, 2010) and Iacus and La Torre (2005a, 2005b). Table 1 provides the results to eight decimal digits by using data in six continents (Africa, Asia, Australia, Europe, South America, and North

* Corresponding author. Department of Public Policy and Public Choice, University of Eastern Piedmont Amedeo Avogadro, via Cavour 84, I-15121, Alessandria, Italy.
E-mail addresses: simone.marsiglio@sp.unipmn.it (S. Marsiglio), davide.latorre@unimi.it (D. La Torre).

[1] According to up-to-date demographic forecasts of the United Nations, the world population annual growth rate is expected to fall gradually from 1.8% (1950–2000) to 0.9% (2000–2050), before reaching a value of 0.2% between the years 2050 and 2100.

[2] See also La Torre (2003), La Torre and Rocca (2003,2005); La Torre and Vrscay (2009) for further details on the fractal-based methods.

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Table 1
Parameters' estimations of population growth over the period 1870–2008.

	g_0	g_1	g_2	g_3
Africa	−0.00763537	0.00000018	−0.00000000	0.00000000
Asia	−0.02926752	0.00000005	−0.00000000	0.00000000
Australia	0.03003342	−0.00000383	0.00000000	−0.00000000
Europe	0.12968633	−0.00000070	0.00000000	−0.00000000
South America	0.00203530	0.00000025	−0.00000000	0.00000000
North America	0.03432962	−0.00000030	0.00000000	−0.00000000

America) over the period 1870–2008. A good fitting curve for Australia, Europe, and North America for this data is the logistic one while South America shows an exponential behavior (g_0, $g_1>0$). Africa and Asia show a negative coefficient g_0 which can be justified in terms of migration effects.

Accinelli and Brida (2005) firstly introduce non-exponential population growth in a growth model, assuming that population dynamics is described by a logistic function. After this work a growing literature studying how different demographic change functions modify standard growth models arises. For example, the Solow model has been extensively analyzed assuming different demographic dynamics. Guerrini (2006) and Brida and Pereyra (2008) introduce respectively bounded population growth (which represents a generalization of the logistic case) and a decreasing population growth in the Solow–Swan model; Bucci and Guerrini (2009) instead study its transitional dynamics in the case of AK technology and logistic population. Also the Ramsey model has been recently extended to encompass several types of population change functions. Brida and Accinelli (2007) study the case of logistic population growth while Guerrini (2009 and references therein) analyzes the case where population growth is given by a bounded function, both in the neoclassical and endogenous framework.

However, all these papers also relax an important standard assumption of optimal growth theory, namely the social welfare function is founded on the Benthamite criterion (total utilitarianism). This criterion says that total welfare is the sum of per-capita welfare across population (the product between population size and average welfare if no heterogeneity among agents is present). These papers[3] instead assume the social welfare function is based on the Millian criterion (average utilitarianism): total welfare equals average welfare or per-capita utility (see Marsiglio, 2010, for a discussion of the implications of both criteria). Such a criterion has been used in order to limit population size and in an optimal theory of growth seems to be somehow reductive. In fact, the main difference in the model's outcome is the effect of population growth on the per-capita consumption dynamics: the Benthamite criterion implies that consumption growth is independent of population dynamics, while the opposite is true for the Millian criterion.

Some papers in the literature discuss how the choice of total rather than average utilitarianism affects the outcome of the model. Such an issue has always been studied in a context of exponential population change, where the general conclusion is that the Benthamite and Millian criteria lead to different effects of population growth on economic performance. This issue is quite popular in the framework of endogenous fertility, in which the steady state outcome is represented by exponential population growth. For example, Nerlove et al. (1982, 1985) and Barro and Becker (1989) analyze a neoclassical setup while Palivos and Yip (1993) an endogenous growth context. Barro and Becker (1989) show that according to the degree of altruism towards future generations, the social welfare function results to be a mix of the Benthamite and Millian criteria. Palivos and Yip (1993) show instead that the Benthamite principle leads to a higher economic growth and a smaller population size. Few papers tackle the issue when population change is exogenous, namely Strulik (2005) and Bucci (2008). They both study the effect of exogenous population growth on the economic growth rate in an endogenous growth model driven by R&D activity, as the degree of agents' altruism towards future generation changes. They both show that the impact of demographic change on the economy varies as the magnitude of the altruism parameter does so. All these works assume population growth is exponential (at least in steady state) and suggest that different utilitarian criteria affect the economic growth rate.

The aim of this paper is studying the introduction of not exponential population change in endogenous growth models, and analyzing the effect of different utilitarian criteria on the model's outcome. We formalize demographic growth as a generic function of population size, discussing how different shapes affect the model. We focus our analysis on a two-sector model of endogenous-growth, á-la Uzawa (1965)–Lucas (1988), since, it has never been analyzed in a framework of non-exponential population growth and, as claimed in Boucekkine and Ruiz-Tamarit (2008), it is one of the most studied and interesting endogenous growth models. In Section 2 we introduce the model in its general form, namely we assume population change depends on a generic function of population size and the social welfare function results to be of the Benthamite or Millian type according to the value of a parameter (representing the degree of altruism). Section 3 performs steady state analysis, which is characterized by a balanced growth path or an asymptotic balanced growth path, according to the features of the population growth function. However, we show that independently on the shape of such a function, the economy converges towards its equilibrium along a saddle path. What is affected by its shape is the dimension of the stable manifold, which can be one or two. We also show the utilitarian criterion adopted is irrelevant for the economic growth rate if in steady state population growth is null, as in the case in which population growth is logistic. In Section 4, instead, we show different examples of population growth function which represent particular cases of our general model. In Section 5 we characterize the global dynamics of the model under a particular parametric restriction concerning the altruism parameter, namely in the case it equals both the capital share and the inverse of the intertemporal elasticity of substitution; in Section 6 we show that under the same condition it is possible to find a closed-form solution for the case in which population dynamics is subject to random shocks and show that uncertainty increases on average the stock of (per-capita) physical and human capital. Section 6 as usual concludes.

2. The model

The model is a Uzawa–Lucas model of optimal growth where the representative agent seeks to maximize his welfare subject to the capital and demographic constraints, choosing consumption, c_t, and the rate of investment in physical capital, u_t. The welfare is the infinite discounted sum of the product of the instantaneous utility function (assumed to be iso-elastic, $u(c_t)=\frac{c_t^{1-\sigma}}{1-\sigma}$, where $\sigma>0$) and the population size weighted by the agent's degree of altruism, $N_t^{1-\varepsilon}$, where $\varepsilon\in[0,1]$. The final good is produced combining physical capital, K_t, and the share of human capital allocated to final production, u_tH_t, according to a Cobb–Douglas technology: $Y_t=K_t^{\alpha}(u_tH_t)^{1-\alpha}$, where $0<\alpha<1$ and $u_t\in(0,1)$. Physical capital, K_t, accumulation is given by the difference between production of the final good and consumption activity: $\dot{K}_t=AK_t^{\alpha}(u_tH_t)^{1-\alpha}-c_tN_t$. The law of motion of human capital, H_t, is instead given by production of new human capital: $\dot{H}_t=B(1-u_t)H_t$. We assume for simplicity that physical and human capital do not depreciate over time. Demographic growth instead is given by a generic function of population size: $\dot{N}_t=N_tg(N_t)$. The shape of such a function, as we shall later show, results to be irrelevant for the equilibrium of the model; the transitional dynamics instead is differently affected by the fact that $g(\cdot)$ shows or not one or more zeros.

The social planner maximizes the social welfare function, that is it has to choose c_t and u_t, in order to maximize agents lifetime utility

[3] An exception is represented by La Torre and Marsiglio (2010). They introduce logistic population growth in a three sectors Uzawa–Lucas (1988) type growth model, in which the welfare function is defined according to the Benthamite criterion. However, since their goal is to focus on endogenous technical progress, they do not study population dynamics (because population size in steady state is constant, under the logistic assumption).