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Public debt in a basic endogenous growth model☆

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ABSTRACT

We work out the mechanism that makes public debt affect the allocation of resources in the long-run. To do so we analyze an *AK* growth model with elastic labor supply and a government sector. The government levies a distortionary income tax and issues bonds to finance lump-sum transfers and non-distortionary public spending. We show that the long-run growth rate is the smaller the higher the debt ratio if the government adjusts public spending to fulfill its inter-temporal budget constraint. If the government adjusts lump-sum transfers the public debt ratio does not affect the balanced growth rate.

1. Introduction

The current debt crisis in Europe demonstrates how unsound fiscal policies can affect the evolution of market economies. Even if a bankruptcy of Greece could be avoided up to now by a joint action of the International Monetary Fund (IMF) and countries of the euro area, the Greek government had to undertake painful measures in order to receive loans from the IMF and from euro area countries. But Greece is not the only country that has drastically suffered from the financial crises that turned into a public debt crisis. Other countries have also been confronted with strong increases in their public debt to GDP ratios that has cast doubt on sustainability of their public finances, such as Ireland, Italy, Portugal and Spain, for example.

These examples make clear that public debt can fundamentally affect the evolution of market economies and the question of how public debt influences an economy arises. A first answer to that question is provided by the Ricardian equivalence theorem stating that a rise in public debt today must be accompanied by an equivalent increase of taxes in the future, in present value terms, so that it is irrelevant whether a given stream of public spending is financed by deficits or by taxes. But the relevance of that theorem for real world economies is rather limited since it is based on very restrictive assumptions, such as lump-sum taxes and no GDP growth for example.

Nevertheless, the Ricardian equivalence theorem contains an important aspect of public finance namely the inter-temporal budget constraint of the government. Hence, in order to guarantee solvency higher public debt today must go along with a corresponding increase of future primary surpluses of the government. The latter can be achieved either through higher taxes, by a reduction of public spending or by higher tax revenues resulting from a rise in GDP. As a consequence, the primary surplus should become a function that positively depends on public debt.

In this contribution we intend to contribute to the research on public debt and economic growth by analyzing a basic endogenous growth model where we allow for public deficits and public debt. Starting point of our paper is the *AK* endogenous growth model with elastic labor supply where we integrate public debt and deficits. Allowing for endogenous labor supply seems to be appropriate since growth models deal with the long-run evolution of economies and the labor supply that may be fixed for the household in the short-run becomes a choice variable in the long-run. Note also that the growth rate of the *AK* growth model would only depend on parameters if the labor supply was given exogenously such that the debt policy of country would never have any effects as regards economic growth. Concerning public spending we assume that it is neither productive nor welfare enhancing. The reason for that assumption is that we are interested in effects of public debt per se holding all other distortions constant. Hence, we fix the distortionary income tax rate so that adjustments of the primary surplus are obtained either with less non-distortionary public spending or with less public lump-sum transfers. We show that the long-run growth rate is the smaller the higher the debt ratio if the government adjusts public spending to meet its inter-temporal budget constraint. If the government adjusts lump-sum transfers to fulfill its inter-temporal budget constraint the public debt ratio does not affect the balanced growth rate.

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Effects of public debt with productive public spending that is distortionary have been analyzed by Yakita (2008) in an OLG model of endogenous growth. Yakita demonstrates that there exists an upper bound for the level of public debt beyond which a sustainable debt policy is excluded with that bound being the larger the higher the stock of productive public capital. Bräuninger (2005) also finds that public debt policy is unsustainable once a critical value of public debt is crossed which, however, in his model only depends on the stock of private capital since he does not consider productive public spending. In the approach by Greiner (2008), who considers an endogenous growth model with infinitely lived agents and productive public capital, it is demonstrated that a higher ratio of public debt to GDP requires resources that reduce public spending so that higher debt leads to lower growth. But, once unemployment is allowed for, this result changes as has been pointed out by Greiner and Flaschel (2010). Then, an increase in deficit financed public spending may generate a higher long-run growth rate. Kamiguchi and Tamai (in press) analyze the endogenous growth model with productive public spending as a flow and public debt and show that balanced growth and sustainability are compatible only if the government fixes the debt to GDP ratio within certain bounds and that the growth rate is the higher the smaller the debt ratio. In Greiner (2011a) public spending is incorporated into the utility function and yields utility for the household sector. With this assumption it is shown that a debt policy that leads to higher long-run growth does not necessarily also imply higher welfare. Finally, Futagami et al. (2008) show that public debt policy may not only affect economic growth but can also be decisive as regards transition dynamics to the balanced growth path and with respect to the emergence of multiple growth paths.

The rest of the paper proceeds as follows. In the next section we present the structure of our growth model which is analyzed in Section 3 and Section 4 concludes.

2. The *AK* endogenous growth model with elastic labor supply and public debt

2.1. Households

The household sector consists of many identical households of mass one and where each household has measure zero. One household is representative for the whole household sector and maximizes the discounted stream of utility. The representative household maximizes its inter-temporal utility (cf. Benhabib and Farmer, 1994) subject to its budget constraint[1]:

$$\max_{C,L} \int_0^\infty e^{-\rho t}\left(\ln C - L^{1+\gamma}/(1+\gamma)\right)dt, \tag{1}$$

subject to

$$(1-\tau)(wL+rW)+T_p = \dot{W}+C, \tag{2}$$

with $\rho\in(0,1)$ the rate of time preference, $\gamma\geq 0$ denotes the inverse of the elasticity of labor supply, w stands for the wage rate and r is the return to wealth $W:=B+K$, which consists of public debt, B, and of physical capital, K, where we neglect depreciation. The variable $C(t)$ denotes per-capita consumption, $L(t)$ gives labor supply and total population is constant. Finally, $\tau\in(0,1)$ is the income tax rate, T_p gives lump-sum transfers and the dot over a variable stands for the time derivative.

To solve this problem we formulate the current-value Hamiltonian which is written as

$$\mathcal{H} = \ln C - L^{1+\gamma}/(1+\gamma) + \lambda\left((1-\tau)(wL+rW)+T_p - C\right) \tag{3}$$

with λ the shadow price of private wealth. Necessary optimality conditions are

$$C = w(1-\tau)L^{-\gamma} \tag{4}$$

$$\dot{C} = C((1-\tau)r-\rho) \tag{5}$$

If the limiting transversality condition $\lim_{t\to\infty} e^{-\rho t}W/C = 0$ holds, the conditions are also sufficient.

2.2. Firms

The productive sector can be represented by one firm which behaves competitively and which maximizes static profits. The production function of the representative firm is given by the following equation,

$$Y = K^{1-\alpha}\bar{K}^{\xi}L^{\alpha}, \tag{6}$$

with $(1-\alpha)\in(0,1)$ the capital share and $\alpha\in(0,1)$ the labor share. The variable Y denotes output and $\bar{K}$ gives the average economy-wide level of capital. We assume constant returns to capital in the economy such that $(1-\alpha)+\xi=1$ holds. Using that $K=\bar{K}$ in the aggregate economy, profit maximization leads to the following conditions,

$$r = (1-\alpha)L^{\alpha} \tag{7}$$

$$w = \alpha L^{\alpha-1}K \tag{8}$$

2.3. The government

The period budget constraint of the government is written as,

$$\dot{B} = rB(1-\tau) - \left(\tau Y - c_p Y - t_p Y\right) = rB(1-\tau) - S, \tag{9}$$

where S is the primary surplus, $C_p = c_p Y$ stands for non-distortionary public spending and $T_p = t_p Y$ gives lump-sum transfers, with $c_p + t_p < \tau$, $0 < c_p, t_p < 1$.

Further, the government sets the primary surplus according to,

$$S = \phi\, Y + \beta\, B, \tag{10}$$

with $\beta\in\mathbb{R}_{++}$ determining how strong the primary surplus reacts to changes in public debt and $\phi\in\mathbb{R}$ determines whether the level of the primary surplus rises or falls with a higher GDP. Since the coefficient β determines how strong the primary surplus reacts to variations of public debt, relative to GDP respectively, we call this coefficient the reaction coefficient.

The motivation for the rule specified in Eq. (10) is that it guarantees that the government fulfills its inter-temporal budget constraint,

$$B(0) = \int_0^\infty e^{-\int_0^\mu (1-\tau)r(\nu)d\nu} S(\mu)d\mu \Leftrightarrow \lim_{t\to\infty} e^{-\int_0^t (1-\tau)r(\mu)d\mu} B(t) = 0$$

on a balanced growth path, see e.g. Greiner (2011b) for details. From an economic point of view this rule simply states that the government raises the primary surplus as public debt rises.

Here, it should also be mentioned that considerable effort has been devoted to estimating (10) empirically. For example, Bohn (1998) has obtained positive values for the reaction coefficient β for the USA by estimating (10) with OLS. Greiner et al. (2007) have performed the same estimation with data for euro area countries and also obtained statistically significant and positive values for β. In Fincke and Greiner (2011) time varying parameters have been estimated and it has been

[1] We omit the time argument t if no ambiguity arises.