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Application of factor models for the identification of countries sharing international reference-cycles☆

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ABSTRACT

Using the methodology developed in Stock and Watson (2002a), this paper proposes to exploit the information that contains the factor loading to identify the countries sharing common factors. The proposal is illustrated by analyzing the relation with the international reference-cycle of a large sample of advanced countries from 1950 until 2006.

1. Introduction

One of the main objectives in the analysis of business cycles is to determine the existence of similar fluctuations in macroeconomic aggregates and whether these fluctuations show substantial synchronization across countries. Recent studies provide evidence that there are many cross-country links in macroeconomic fluctuations. This is the case of Artis et al. (2004), Darvas and Szapáry (2004), Camacho et al. (2006), Krolzing and Toro (2005), Böwer and Guillemineau (2006) for EU countries and Cancelo and Uriz (2001), Lumsdaine and Prasad (2002), Cotis and Copel (2005) for the OECD, among others.[1]

The idea of summarizing the state of the economy as a synthetic index, or reference cycle, describing the common behavior of economic variables, was introduced by Burns and Mitchell (1946).[2] The methodology used for such purposes is the application of multivariate techniques of the factor analysis that aims to simplify a broad set of observed interdependent data and explain them in terms of a small number of unobserved and independent factors, with the lowest loss of information possible. This methodology has been improved by introducing dynamics in the traditional factor model (Geweke, 1977 and Sargent and Sims, 1977), as it is considered that the contemporary relationship between economic variables, without considering the delays, can only be interpreted as a reference cycle (Stock and Watson, 1989).

The implementation of dynamic factor analysis methodology is not always possible, especially when the number of countries under study is large. Two approaches have been developed in the literature to overcome this limitation: the first is an approximation to the dynamic factor model that is closer to multivariate statistical analysis. This approach follows the research of Stock and Watson (1998, 2002a), which has been extended in subsequent articles by Stock and Watson (1999, 2002b) for the United States, Camacho and Sancho (2003) for Spain, Artis et al. (2001) for the UK, Gosselin and Tkacz (2001) for Canada, and Angelini et al. (2001) for the euro area.

A second methodology is based on estimates from a Bayesian approach, where we can find references in Otrok and Whiteman (1998) who have subsequently sought to generalize and extend the work of Kose et al. (2003a) and Kose et al. (2003b, 2005) and Kose et al. (2008) for an international cycle. This approach resolves some of the limitations of traditional non-parametric analysis, but it raises other questions such as the fact that it is not possible to test some do the assumptions related to the dynamics between countries imposed in the model.

A common aspect in the papers that have applied these methodologies to estimate the international business cycles is that

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[1] See de Haan (2008) for a survey of this literature.

[2] This is the approach behind the coincident indicator of the NBER.

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the sample of countries employed in their analysis has traditionally been assumed and they do not consider the possibility of testing if all the countries share this behavior. Unlike these works, the objective of this paper is to estimate the international reference cycles and identify countries sharing this behavior. This goal is achieved by estimating a factor model that allows the factor loadings to contain the meaning of the relevant weight in relation to the factor. This information, will make possible that the common factors and the countries consider in the analysis will be selected by the model. Therefore, we are offering a criterion to determine which countries share the common international fluctuation and to exclude those which would only introduce noise in the model. We have applied this proposal to a large sample of advanced countries and the estimate is carried out in terms of the economic cycle (growth rate) of GDP per capita on an annual basis from 1950 to 2006.

The present paper is organized as follows: Section 2 summarizes the factor model methodology for large samples. Section 3 presents and analyzes the results of the empirical application of the proposal and Section 4 concludes.

2. Factor model methodology for large samples

In this research we follow the methodology developed in Stock and Watson (1998, 2002a) as a main reference for the factor analysis of business cycles. The statistical method used by these authors is based on a priori assumption that finite lags in the representations of the variables under study, depending on latent factors (not observable), make it possible to use a principal component analysis in the estimation of the factors. The use of principal components facilitates the factor estimation when the number of explanatory variables is large (situations that are becoming frequent with the development of large databases) and it can cope with irregularities in the data, using information from series with missing values.

Formally, in the model it is assumed that a set of variables X_t can be represented by the common latent factors, such as:

$$X_t = \Lambda_t F_t + e_t; \tag{2.1}$$

where the idiosyncratic disturbances, $e_t = (e_{1,t}, e_{2,t}, \ldots, e_{N,t})'$, contained in a $N\times 1$ vector, are serially correlated and slightly cross-sectionally correlated with other variables in the model. In addition, the factor loadings vary over time according to:

$$\Lambda_t = \Lambda_{t-1} + h\xi_t; \tag{2.2}$$

where h is the diagonal $N\times N$ scaling matrix, and ξ_t are $N\times r$ stochastic disturbances.

For the estimation of the model (2.1)–(2.2), when N is large, Stock and Watson (1998, 2002a) propose an estimation approach called quasi-likelihood, since we must make very restrictive assumptions on the model: such that $\Lambda_t = \Lambda$, $h = 0$ and e_{it} is $i.i.d.\ N(0, \sigma_e^2)$ and independent across series. Under these assumptions, the maximum likelihood estimator for (Λ, F) gives consistent estimates of the factors and factor loadings, by solving a problem of nonlinear least squares:

$$V\left(\tilde{F}, \tilde{\Lambda}\right) = (NT)^{-1} \sum_{i=1}^{N} \sum_{t=1}^{T} \left(x_{it} - \tilde{\lambda}_i \tilde{F}_t\right)^2; \tag{2.3}$$

So that Eq. (2.3) is the objective function of the supposed values of the factors, $\tilde{F}_t = \left(\tilde{F}_1, \ldots \tilde{F}_T\right)'$, and loadings, $\tilde{\Lambda}$, where $\hat{F}$ and $\hat{\Lambda}$ are the values resulting from the efficient minimization. In practice, to solve this problem we find two cases, depending on the number of variables (N) and the number of periods (T): $N>T$ (Situation A) and $N<T$ (Situation B). These two situations are described in Stock and Watson (2002a) in order to offer viable solutions to solve them statistically. For this purpose, we chose the solution that allows the factor loadings to be interpreted as the weight with respect to the common factor (Situation B). Thus, we obtain useful information in order to identify countries sharing the international business cycle.[3]

So the options are:

Situation A: When the sample is large and $N>T$:

In this case, a less computationally complex approach is required, since the determinant of the matrix $X'X$ is zero and there is therefore no single solution. To this end, Stock and Watson (2002a) propose the use of the matrix XX' $(N\times N)$, which implies maximizing $tr\left[\tilde{F}XX'\tilde{F}\right]$ subject to $\tilde{F}\tilde{F}/T = I_r$. Therefore, from the matrix XX', in vector notation:

$$\frac{1}{T}\sum_{i=1}^{T} x_t x_t'. \tag{2.4}$$

Consequently, the estimates of the first k eigenvectors are directly the principal components and therefore the factors.

However, if one analyzes the content of the matrix XX':

$$XX' = \begin{pmatrix} \sum_{i=1}^{N} x_{i,1}^2 & \sum_{i=1}^{N} x_{i,1}x_{i,2} & \cdots & \sum_{i=1}^{N} x_{i,1}x_{i,T} \\ \sum_{i=1}^{N} x_{i,2}x_{i,1} & \sum_{i=1}^{N} x_{i,2}^2 & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^{N} x_{i,T}x_{i,1} & \cdots & \cdots & \sum_{i=1}^{N} x_{i,T}^2 \end{pmatrix}. \tag{2.5}$$

The main diagonal of the matrix shows the variances between the data, considering only one year for each country. Meanwhile, outside the main diagonal is the sum of serial correlations of each country and not includes correlations between countries, which are the most important information to carry out the factor analysis.

Situation B: When the sample is large and $N<T$:

The minimization problem is equivalent to maximizing $tr\left[\tilde{\Lambda}X'X\tilde{\Lambda}\right]$ subject to $\tilde{\Lambda}\tilde{\Lambda}/N = I_r$; the solution of which leads to the principal component analysis, whose detailed description can be reviewed in Peña (2002). The factors are selected using the appropriate eigenvectors of k largest eigenvalues of the matrix $T\times T$:

$$\frac{1}{N}\sum_{i=1}^{N} x_i x_i'; \tag{2.6}$$

which is the covariance matrix of X (where the matrix has variances of each series on the main diagonal – interpretable factor loadings – and contemporary correlations between countries that are off the diagonal). This computing proposal is equivalent to the static factor model, since only the contemporary relationship between the countries are considered, so that we would be referring to the so-called diffusion index of Stock and Watson (1989, 1998) for the reference cycle.

Therefore, in this paper the methodological proposal will be to implement the static factor analysis for the sample of countries ($N<T$) in two stages: 1) Estimate the model with all countries in the sample, selecting the corresponding number of factors according to the model

[3] Another possible application of this information is to the analysis of factor loadings stability (see Cendejas et al., 2011).