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Endogenous fluctuations induced by nonlinear pollution accumulation in an OLG economy and the bifurcation control ☆

Dong Cao [a,*], Lin Wang [b], Yaozhong Wang [c]

[a] School of Economics and Management, Xidian University, Xian, Shaanxi, 710071, PR China
[b] Department of Mathematics and Statistics, University of New Brunswick, Fredericton, NB, Canada
[c] College of Economics and Trade, Hunan University, Changsha, Hunan, 410079, PR China

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ABSTRACT

This paper aims to study the influence of nonlinear pollution accumulation on economic growth by incorporating the nonlinear pollution accumulation into an OLG model. It is shown that the nonlinearity can yield very complex dynamics, including the flip bifurcation resulting in sustained fluctuations in economy. This indicates that the nonlinear pollution accumulation can be a source of intergenerational inequity. To stabilize the complex motion and thus to control the fluctuations, a bifurcation control method from control theory is proposed. The welfare analysis shows the sustainable development criterion can be met in the controlled system but not in the original system.

1. Introduction

Economic growth process and governments' impacts on growth rates have been studied extensively for a long time (see, for example, [1–3]), while until recent decades, more attention has been paid to study the relationship between economic growth and environmental quality [4]. In literature, there are three major different views on the relation between economic growth and the environment: 1) growing economic activity will bring even greater harm to the environment [5]; 2) the environment is being along with the economic growth as higher incomes increase demand for goods and services which are less material intensive [6]; 3) there exists an inverted U-shaped "environmental Kuznets curve (EKC)" [7–10]. The EKC theory suggests that economic inequality increases over time while a country is developing, and then after a certain average income is attained, inequality begins to decrease.

There does have some evidence showing the EKC applies to a selected set of pollutants such as urban air pollution, the oxygen regime in river basins, fecal contamination of river basins, and contamination of river basins by heavy metals [11]. As the EKC hypothesis assumes that people will invest more to improve the environment quality and to reduce pollution when their incomes pass a certain value, there are still some reasons to be very cautious in applying the curve to study the environment quality. The Nobel Prize winner in economics Arrow et al. in [12] argued from four aspects. First, it is valid for pollutants involving neither local short term nor long term costs. Second, it has not considered the emissions of pollutants and resource stocks. Third, there is no caution about the consequence of emission reductions. Fourth, emission reduction due to institution reforms which may ignore international costs between generations. This suggests that the environmental consequences of growing economic activities may, accordingly, be very complex.

In this paper, we investigate the complex relation between economic growth and the environment. More specifically we aim to understand the influence of nonlinear pollution accumulation on the growth of economy. To this end, we incorporate the nonlinear pollution accumulation and scale technology into an overlapping generation (OLG) model and then we obtain the conditions under which a flip bifurcation occurs. The occurrence of flip bifurcation indicates the instability and cyclical behavior (business cycles) in the economic system. Such unstable fluctuation behavior, in an economic system is often an unfavorable phenomenon. To stabilize the economic system, we then propose a delayed feedback control (DFC) method to reduce the fluctuations. Welfare analysis shows that the controlled system conforms with the sustainable development criterion (SDC) while the uncontrolled system does not.

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* Corresponding author at: School of Economics and Management, Xidian University, Xian, Shaanxi, 710071, PR China.
E-mail addresses: ivycd0313@hotmail.com (D. Cao), lwang2@unb.ca (L. Wang), yzhwang@hnu.cn (Y. Wang).

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The rest of the paper is organized as follows. In Section 2, we present the basic OLG model with nonlinear pollution accumulation and scale technology and we study the model's dynamics. A bifurcation control model designed for the OLG model based on DFC is introduced in Section 3. The welfare analysis is carried out in Section 4 and in Section 5 we conclude the paper.

2. The basic model

We consider an OLG economy and assume that the population of generation $t(t=0, 1, \cdots, \infty)$ is constant and normalized to one. Every individual lives for two periods t (youth) and $t+1$ (old age). The young generations supply one unit of labor inelastically and share the income w_t between the purchase of productive capital k_{t+1} and the abatement of pollution emissions d_t. The old generations obtain utility from consumption C_{t+1} and environmental quality. In addition, we assume that the old generations rent their capital to firms, which are remunerated at the real interest rent r_{t+1}.

2.1. The nonlinear pollution accumulation

As in [4], pollution is assumed to accumulate according to the equation given by

$$A_{t+1} = (1-m(A_t))A_t + ak_t - bd_t \tag{1}$$

where $a>0$ is the emission rate of pollution per unit of capital; $b>0$ is the coefficient of the linear abatement technology; $A_t>0$ can be understood as the stock of pollution inherited from the previous period; $m(A_t)\in(0,1)$ is the natural rate of pollution absorption. Following the line in [13], we assume that

$$m(A_t) = 1 - \frac{A_t}{\hat{A}} \tag{2}$$

where $\hat{A}$ represents the maximum anthropogenic additions to atmospheric concentrations for which non-atmospheric sinks and storage continues to remove a portion of emissions. When there is no emission from human activity ($A_t=0$), the natural system is in equilibrium at its pre-industrial level and the uptake is complete ($m=1$). Thus the nonlinear pollution accumulation is described by

$$A_{t+1} = \frac{A_t^2}{\hat{A}} + ak_t - bd_t \tag{3}$$

2.2. The consumers

Each agent maximizes its lifetime utility, which depends on consumption and environmental quality. The problem facing the representative agent is given as follows

$$max \quad C_{t+1} - \frac{A_{t+1}^2}{2B} \tag{4}$$

subject to the constraints

$$k_{t+1} + d_t = w_t \tag{5}$$

$$C_{t+1} = r_{t+1}k_{t+1} \tag{6}$$

and Eq. (3).

Here Eq. (4) gives the lifetime utility function with the old generations obtain utility from consumption and environmental quality. Consumers' preferences are given by the utility function $C_{t+1} - \frac{A_{t+1}^2}{2B}$, where $B>0$ is a scaling parameter. Eqs. (5) and (6) are the budget constraints in the first and second periods. Since people consume in both periods of life, they have to pay for consumption in the second period by saving in the first period.

Then the first order condition with respect to d_t gives

$$r_{t+1} = A_{t+1} \cdot \frac{b}{B} \tag{7}$$

2.3. The representative firm

We suppose that the final good is produced by the representative firm with a constant return to scale technology. Thus the production function can be expressed in intensive form as

$$y = f(k) \tag{8}$$

where $k \equiv \frac{K}{L}$ is capital per worker, $y \equiv \frac{Y}{L}$ is output per worker. The per capital production is assumed to be a continuous function satisfying the standard conditions

$$f'(k_t) > 0, f''(k_t) < 0 \ for \ k_t > 0, \text{ and } f(1) = 1 \tag{9}$$

The marginal products of the factor inputs are given by

$$MPK = \frac{\partial Y}{\partial K} = f'(k) \tag{10}$$

$$MPL = \frac{\partial Y}{\partial L} = f(k) - k \cdot f'(k) \tag{11}$$

The standard maximization of profit by competitive firms leads to the equation of net marginal products to factor prices

$$r_t = f'(k_t) \equiv r(k_t), \quad \text{and} \quad w_t = f(k_t) - k_t f'(k_t) \equiv w(k_t) \tag{12}$$

There are some useful relationships. First, the capital share of income is defined as $s(k) = \frac{f'(k)k}{f(k)} \in (0,1)$. Moreover, we note that the elasticities of wage and interest rate with respect to k could be defined as the percentage change in wage and interest rate caused by a 1 percent change in the capital: $\varepsilon_w(k) = \frac{w'(k)k}{w(k)}$ and $\varepsilon_r(k) = \frac{r'(k)k}{r(k)}$. Finally, the elasticity of capital-labor substitution $\sigma(k)$ is determined by

$$\frac{1}{\sigma(k)} = \frac{d \ ln \frac{w(k)}{r(k)}}{d \ ln(k)} = \varepsilon_w(k) - \varepsilon_r(k) \tag{13}$$

This quantity measures the extent to which firms can substitute capital from labors as the relative productivity or the relative cost of the two factors changes. And we can get the substitution elasticity is depends on the difference between the elasticities of wage and interest rate.

By using the second part of Eq. (12), we get $f(k) = kr(k) + w(k)$ and $w'(k) = -kr'(k)$. From this last relation, we obtain

$$\varepsilon_w(k) = \frac{s(k)}{\sigma(k)}, \quad and \quad \varepsilon_r(k) = -\frac{1-s(k)}{\sigma(k)} \tag{14}$$

The first equation of Eq. (14) shows that the proportion of the capital share of income $s(k)$ in the elasticity of capital-labor substitution $\sigma(k)$ is exactly the elasticity of wage ε_w.

2.4. Local dynamics

It follows from Eq. (3), (5), (7) and (12) that the system is given by

$$B\frac{f'(k_{t+1})}{b} - bk_{t+1} = B^2\frac{[f'(k_t)]^2}{b^2\hat{A}} + ak_t - b[f(k_t) - k_t f'(k_t)] \tag{15}$$