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On the application of the rank tests for nonlinear cointegration to PPP: The case of Papua New Guinea ☆

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ABSTRACT

Much interest has been paid recently to the nonlinear cointegrating relations existing among economic variables. Various testing procedures are already available to test for the existence of nonlinear cointegration. For example, Breitung (2001) proposes rank tests and his testing procedure has been broadly applied. In this study, we warn against a blind application of the rank cointegration tests, particularly to economic variables that evidence certain behavior. As an illustration, we employ the nominal exchange rates and relative prices of Papua New Guinea against her major trading partners with the objective of testing the validity of purchasing power parity for the country. Our simulation results also confirm our warnings. Additionally, we provide some simple solutions to the problem we encounter herein.

1. Introduction

In the seminal paper on cointegration of Engle and Granger (1987), only the linear cointegrating relation is considered. Recently, a great deal of attention has been focused on modeling nonlinear cointegrating relations. For instance, Karlsen et al. (2007), Schienle (2008), and Wang and Phillips (2009a, 2009b, 2011) have developed an asymptotic theory for the nonparametric kernel regression of nonlinear cointegrated systems. Additionally, Breitung (2001) has proposed rank tests for nonlinear cointegration. By way of contrast with other approaches, see for example the studies of Choi and Saikkonen (2004, 2010), Saikkonen and Choi (2004), Marmer (2008), Hong and Phillips (2010), and Kasparis and Phillips (2009), the rank tests do not require that any functional forms be specified in advance. The tests are based on the rank transformation of the data series, and are known to be robust to outliers. The rank tests have been applied broadly to certain economic variables. For example, Haug and Basher (2011), Li (2006), and Liew et al. (2009, 2010), among others, have applied these tests to real exchange rates and international stock price indices.

In this study, we warn against a blind application of the rank tests to economic data series that evidence certain behaviors. Whereas the concerns we raise here can be readily inferred from the results of Breitung (2001), it appears worthwhile to provide some specific examples in which the rank cointegration tests may seriously lack power.

To illustrate our concerns, we employ the quarterly nominal exchange rates and relative prices of Papua New Guinea with her four major trading partners, and assess the validity of purchasing power parity (PPP) for the country. Because the country had experienced persistent depreciations and higher inflation than her trading partners during the sample period, its nominal exchange rate and relative prices moved into different directions. This feature in the Papua New Guinea data provides a unique opportunity to raise our concerns regarding the rank tests. We call this phenomenon a “rank problem,” and more details will be provided in Section 4. Simple simulations also demonstrate that for certain data generating processes, the rank tests exhibit very low power. The problem is intrinsic to the rank tests, owing to the way they are constructed, and it appears that there are, in general, no other ways to avoid it. For the particular case discussed in this study in the context of PPP, it turns out that some simple solutions are indeed available. Overall, our results should alarm future users of these rank tests for certain economic data series.

In the following section, we present a brief review of the rank tests for nonlinear cointegration proposed by Breitung (2001).

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2. Rank tests for nonlinear cointegration

Assume that variables x_t and y_t have the following relation:

$$u_t = g(y_t) - f(x_t) \tag{1}$$

for $t=1,\ldots,T$, where $g(y_t) \sim I(1), f(x_t) \sim I(1)$, and $u_t \sim I(0)$. T denotes the sample size. Given that $u_t \sim I(0)$, x_t and y_t are nonlinearly cointegrated. We assume that the functions $f(x_t)$ and $g(y_t)$ increase monotonically. When it is uncertain as to whether or not these functions are monotonically increasing, a two-sided test will be presented below. No explicit functional forms must be specified, which contrasts with the other nonlinear cointegration tests referenced in the previous section.

Define the ranked series as $R_T(y_t) = \text{Rank of } [y_t \text{ among } y_1,\ldots,y_T]$ and construct $R_T(x_t)$ accordingly. The rank test is constructed by replacing $f(x_t)$ and $g(y_t)$ with the ranked series. Because $f(x_t)$ and $g(y_t)$ increase monotonically, a sequence of ranks is invariant to a monotonic transformation of the data. Therefore, we have

$$R_T[g(y_t)] = R_T(y_t)$$

and

$$R_T[f(x_t)] = R_T(x_t).$$

Define the rank difference as

$$d_t = R_T(y_t) - R_T(x_t). \tag{2}$$

Consider the following distance measures between the sequences $R_T(y_t)$ and $R_T(x_t)$:

$$\kappa_T = T^{-1} \sup_t |d_t| \tag{3}$$

and

$$\xi_T = T^{-3} \sum_{t=1}^{T} d_t^2. \tag{4}$$

When $f(x_t)$ and $g(y_t)$ move together, d_t should be small. Therefore, the null hypothesis of no (nonlinear) cointegration is rejected if the test statistics are too small. In fact, under the alternative hypothesis of a cointegrating relationship, both κ_t and ξ_t converge to 0 as $T \to \infty$, and thus the tests are consistent.

We demonstrate, however, that when $f(x_t)$ and $g(y_t)$ move into *different* directions, d_t could be large, even though they are still cointegrated. While the cointegrating coefficients are routinely found to be positive in many applications, we note for the data series to be examined here that the cointegrating coefficient – if it exists – should be negative.[3] In this case, the application of the rank tests could generate quite unreliable results.

When $f(x_t)$ and $g(y_t)$ are correlated, the test statistics are corrected with the estimated correlation coefficient of rank differences. For instance,

$$\kappa_T^* = \frac{\kappa_T}{\hat{\sigma}_{\Delta d}} \text{ and } \xi_T^* = \frac{\xi_T}{\hat{\sigma}_{\Delta d}^2} \tag{5}$$

where

$$\hat{\sigma}_{\Delta d}^2 = T^{-2} \sum_{t=2}^{T} (d_t - d_{t-1})^2.$$

[3] In the cointegrating regression $y_t = \alpha + \beta x_t + \varepsilon_t$, β is a cointegrating coefficient and $(1, -\beta)$ is a cointegrating vector.

The simulation results in Breitung's (2001) study demonstrate that the κ_T^* and ξ_T^* tests should be applied when the correlation between $f(x_t)$ and $g(y_t)$ is small. Of course, we have only x_t and y_t, and thus the correlation cannot be estimated. Additionally, Breitung (2001) defines

$$\kappa_T^{**} = \frac{\kappa_T^*}{\lambda_\kappa^\alpha(E\rho_T^R)} \text{ and } \xi_T^{**} = \frac{\xi_T^*}{\lambda_\xi^\alpha(E\rho_T^R)} \tag{6}$$

and suggests that $\lambda_\kappa^\alpha(E\rho_T^R)$ be approximated with $\lambda_\kappa^{0.05} \simeq 1-0.174(\rho_T^R)^2$ and $\lambda_\xi^\alpha(E\rho_T^R)$ with $\lambda_\xi^{0.05} \simeq 1-0.462\rho_T^R$, where ρ_T^R is the correlation coefficient of the rank differences:

$$\rho_T^R = \frac{\sum_{t=2}^{T} \Delta R_T(x_t) \Delta R_T(y_t)}{\sqrt{\left(\sum_{t=2}^{T} \Delta R_T(x_t)^2\right)\left(\sum_{t=2}^{T} \Delta R_T(y_t)^2\right)}}.$$

Furthermore, it is also possible to test for the existence of cointegration among $k+1$ variables, $y_t, x_{1t},\ldots,x_{kt}$, where it is assumed that $f_j(x_{jt})$ $(j=1,\ldots,k)$ and $g(y_t)$ are monotonic functions. Let $R_T(\mathbf{x_t}) = [R_T(x_{1t}),\ldots,R_T(x_{kt})]'$ be a $k \times 1$ vector and $\tilde{\mathbf{b}}_T$ be the least squares estimate from a regression of $R_T(y_t)$ on $R_T(\mathbf{x_t})$. Using the residuals

$$\tilde{u}_t^R = R_T(y_t) - \mathbf{b}'_{\mathbf{T}} R_T(\mathbf{x_t}) \tag{7}$$

a multivariate rank test statistic is obtained from the normalized sum of squares:

$$\Xi_T[k] = T^{-3} \sum_{t=1}^{T} \left(\tilde{u}_t^R\right)^2.$$

To account for a possible correlation between the series, the modified test statistic should be applied:

$$\Xi_T^*[k] = \frac{\Xi_T[k]}{\hat{\sigma}_{\Delta u}^2} \tag{8}$$

where $\hat{\sigma}_{\Delta u}^2 = T^{-2} \sum_{t=2}^{T} \left(\tilde{u}_t^R - \tilde{u}_{t-1}^R\right)^2$. The critical values of the rank tests are tabulated in Table 1 in Breitung's (2001) study through simulations.

When the tests indicate the existence of cointegration, it is also possible to test whether the relation is linear or nonlinear. The test is called a rank test for neglected nonlinearity. The test statistic follows a standard χ^2 distribution asymptotically under the null hypothesis of linear cointegration. However, as we find no evidence for cointegration for the data series to be discussed in this paper, we omit further discussion regarding the rank test for neglected nonlinearity.

Table 1
Breitung (2001) rank tests for nonlinear cointegration between nominal exchange rates E and relative prices $\frac{p^f}{p^{PNG}}$.

	κ_T	ξ_T	κ_T^*	ξ_T^*	κ_T^{**}	ξ_T^{**}	$\Xi_T^*[1]$
Trading partner							
Australia	0.9701	0.3097	1.0438	0.3585	1.0445	0.3693	0.3312
Japan	0.9701	0.2845	0.9486	0.2720	0.9489	0.2674	0.2264
U.K.	0.9851	0.2720	0.9224	0.2384	0.9235	0.2297	0.2317
U.S.	0.9701	0.2706	1.2449	0.4457	1.2452	0.4384	0.4946
5% c.v.	0.5524	0.0423	0.3635	0.0188	0.3635	0.0188	0.0197
Linearly detrended							
Australia	0.8806	0.1839	0.7392	0.1296	0.7393	0.1283	0.1128
Japan	0.8060	0.1716	0.7197	0.1368	0.7197	0.1367	0.1223
U.K.	0.9403	0.1748	1.0273	0.2086	1.0273	0.2092	0.1835
U.S.	0.9104	0.1585	1.3230	0.3347	1.3245	0.3479	0.2978
5% c.v.	0.7020	0.0665	0.3283	0.0665	0.3284	0.0163	0.0172

Various rank cointegration test results are reported for each trading partner of PNG with the original and linearly detrended data series; see Eq. (10). None of the test statistics is significant, even at a significance level of 10%. c.v.: critical value.