Economic Modelling 28 (2011) 1226–1234

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/ecmod

Testing for a unit root with covariates against nonlinear alternatives

Ching-Chuan Tsong *

Department of Economics, National Chi Nan University, Taiwan

ARTICLE INFO

Article history:
Accepted 12 January 2011

JEL classification:
C32
F31

Keywords:
Unit roots
Covariates
Nonlinear processes
Real exchange rates

ABSTRACT

This paper proposes a new procedure for testing the unit root null against stationary but nonlinear alternatives. This test can be viewed as a generalization of the one developed by Kapetanios et al. (2003) (the KSS test) by incorporating stationary covariates. The asymptotic distribution of the test is derived and the asymptotic critical values are tabulated. A set of Monte Carlo simulations show that our test generally achieves large power improvements over the KSS test. An illustrated empirical application indicates that our proposed test is able to unveil more evidence than the KSS test in favor of no unit root of real exchange rates in 15 Asian countries.

1. Introduction

A large number of unit root tests have been proposed for testing the unit root null against the alternative hypothesis of linear stationarity. While the presence or absence of a unit root has important implications (e.g. see Granger and Newbold, 1974; Diebold and Kilian, 2000), many remain dissatisfied with the standard linear framework. One possible reason is that for a particular macroeconomic variable, such unit root tests have low power and constantly fail to yield strong evidence against the unit-root hypothesis, thereby contradicting the prediction of certain economic theories. A prominent example is the purchasing power parity (PPP) hypothesis—a fundamental assumption for numerous exchange rate determination models.

As noted by Sercu et al. (1995), Sarno and Taylor (2001) and Taylor (2003), the presence of transactions costs of arbitrage between spatially separated markets, tariff and non-tariff barriers as well as market interventions may lead numerous economic variables, including the real exchange rate, to display nonlinear adjustments towards an attractor. Based on this argument, Kapetanios et al. (2003) developed a unit root test, denoted as KSS, which expanded the conventional ADF test by retaining the null hypothesis of nonstationarity against the alternative of stationary but nonlinear exponential smooth transition autoregressive (ESTAR) processes. They showed by simulation that the KSS test is more powerful than the ADF counterpart against the stationary ESTAR processes. Although the KSS test is able to unveil more evidence in favor of no unit root as compared with the ADF test in a large number of empirical studies, it still cannot fully account for the widespread failure to reject the unit-root hypothesis (see e.g. Liew et al., 2004; Bahmani-Oskooee et al., 2008).

Including stationary covariates in the regression equation is another promising approach for improving the power of unit root tests. Based on the notion that a particular time series to be tested is rarely observed in isolation, Hansen (1995) showed that additional information contained in stationary covariates that are correlated with the series can be exploited to obtain the covariate ADF (CADF) test that has higher power than the ADF test. Motivated by Hansen (1995), Elliott and Jansson (2003) and Jansson (2004) also applied this technique for developing tests that are more powerful than their univariate counterparts.[1]

The goal of this paper is to develop a generalization of the KSS test, called the covariate KSS (CKSS) test, which is, in spirit, similar to the CADF generalization of the ADF test by considering the regression model that includes leads and lags of stationary covariates. This study is more advanced as compared with extant literature in the following three senses. First, we derive the asymptotic distribution of the CKSS test, which is not the conventional KSS distribution, but a convex combination of the KSS distribution and the functional of Brownian motions; the combination depends on the nuisance parameter of the correlation between the equation error and regression covariates. The distributions of the CKSS and KSS tests coincide provided the correlation is zero.

* Department of Economics, National Chi Nan University, Nantou 545, Taiwan. Tel.: +886 49 291 0960x4662; fax: +886 49 291 4435.
E-mail address: tcc126@ncnu.edu.tw.

[1] Although the use of panel data is also a common method for increasing the power of the standard unit root tests, it is not relevant to this paper and omitted for brevity.

0264-9993/$ – see front matter
doi:10.1016/j.econmod.2011.01.003

Second, finite sample performance of the CKSS test is examined via a set of simulations, and compared with the existing tests in terms of both finite sample sizes and powers. We find that as the correlation between the equation error and the regression covariates becomes stronger, the CKSS test yields substantial power gains over the KSS test. The results are similar to those in Hansen (1995) for the CADF test versus the ADF test. In addition, the CKSS test can also be regarded as a modified CADF test against the alternative of stationary ESTAR processes. Compared with the CADF test, the CKSS test is more powerful under the alternative hypothesis of stationary ESTAR processes, particularly in the region close to the null where the series are highly persistent.

Third, the CKSS test is applied to examine the empirical validity of the long-run PPP among 15 Asian economies. A majority of these countries are developing countries where the levels of transactions costs and foreign exchange rate interventions may be much higher than those in developed countries, thereby suggesting a case for real exchange rates to possibly exhibit nonlinear adjustment towards PPP. The results of the CKSS test are attractive. It yields much stronger evidence in favor of no unit root, wherein the real exchange rates in 13 cases display long-run mean reversion. In contrast, the KSS test fails to do so and only detects 6 cases that are stationary. The remarkable power gains of the CKSS test over the KSS test through the inclusion of stationary covariates account for these results.

The remainder of this paper is organized as follows. Section 2 presents the assumptions, the testing framework and derives the asymptotic distribution of the CKSS test under the unit root null. Further, asymptotic critical values are also tabulated. Section 3 reports a simulation study on the finite sample properties of the test. Section 4 applies the test to the real exchange rates in 15 Asian economies. Section 5 offers brief concluding remarks. The Appendix A contains mathematical proofs.

2. Unit root test with covariates in the nonlinear framework

2.1. Model and assumptions

We consider the univariate time series (y_t), given by

$$\alpha(L)\Delta y_t = \gamma y_{t-1}G(s_t;\theta,c) + u_t \tag{1}$$

for $t=1,2,\ldots,n$, where $\alpha(L) = 1-\sum_{k=1}^{p}\alpha_k L^k$ is a p-order polynomial in the lag operator L; $G(s_t;\theta,c)$ is a transition function, s_t a transition variable, $\theta \geq 0$ a transition speed parameter representing the speed of transition between different regimes in the data, and c is the location parameter; $\Delta = 1-L$ is the usual difference operator. As in Kapetanios et al. (2003), we set $G(s_t;\theta,c)=1-\exp(-\theta y_{t-1}^2)$ with the transition variable $s_t=y_{t-1}$, and the location parameter $c=0$ (see van Dijk et al., 2002 for further details). In this setting, as $y_{t-1}\to\pm\infty$, $G\to 1$, and as $y_{t-1}\to 0$, $G\to 0$, provided that $\theta>0$. Consequently, the process can be regarded as a regime-switching process that permits two regimes—a middle regime when y_{t-1} is close to zero and an outer regime when y_{t-1} becomes larger (either positive or negative). Moreover, the transition from one regime to the other is smooth. We also permit the regression error (u_t) in Eq. (1) to be related to other stationary variables known as covariates. To be specific, let u_t be generated by

$$u_t = \beta(L)'x_t + \varepsilon_t, \tag{2}$$

where x_t is an $m\times 1$ zero mean stationary covariate vector; $\beta(L) = \sum_{k=-q_1}^{q_2}\beta_k L^k$ is a lag polynomial that permits leads and lags of x_t in the equation of u_t.

Combining Eqs. (1) and (2), we obtain

$$\Delta y_t = \gamma y_{t-1}\left[1-\exp\left(-\theta y_{t-1}^2\right)\right] + \sum_{k=1}^{p}\alpha_k\Delta y_{t-k} + \sum_{k=-q_1}^{q_2}\beta_k' x_{t-k} + \varepsilon_t, \tag{3}$$

which is the model given in Kapetanios et al. (2003) augmented by leads and lags of the m stationary covariates in x_t, provided that $\beta(L)\neq 0$. Note that if θ is positive, then y_t follows a nonlinear but globally stationary process provided that $-2<\gamma<0$, as shown in Kapetanios et al. (2003), which is also assumed to hold. On the other hand, under the condition of $\theta=0$, y_t follows a unit root process.

Kapetanios et al. (2003) developed the KSS test without incorporating any covariates and argued that the KSS test is more powerful than the conventional ADF test when the series to be tested follows stationary but ESTAR dynamics. Furthermore, it is rare that macroeconomic variables are observed in isolation but related to one another to a certain extent. According to Hansen (1995), the CADF test can achieve large power gains over the ADF test by including correlated stationary covariates in the regression. Borrowing similar arguments from Hansen (1995), it may be expected that the inclusion of related covariates will provide useful information to further enhance the power of the KSS test. These motivate us to develop a testing procedure in this context, mainly focusing on the specific parameter θ. More specifically, we consider the test of the unit root null hypothesis:

$$H_0 : \theta = 0 \tag{4}$$

for y_t as in Eq. (3) against the nonlinear stationary alternative hypothesis

$$H_1 : \theta > 0. \tag{5}$$

Our test statistic for testing the unit root in y_t is introduced in the next section.

For the subsequent analysis, define the long-run covariance matrix of $\zeta_t=(u_t,\varepsilon_t)'$ as

$$\Omega = \sum_{k=-\infty}^{\infty}\mathrm{E}(\zeta_t\zeta_{t-k}') = \begin{pmatrix}\sigma_u^2 & \sigma_{u\varepsilon}\\ \sigma_{u\varepsilon} & \sigma_\varepsilon^2\end{pmatrix}, \tag{6}$$

and further define

$$\rho^2 = \frac{\sigma_{u\varepsilon}^2}{\sigma_u^2\sigma_\varepsilon^2}, \tag{7}$$

which is the long-run squared correlation between u_t and ε_t, measuring the relative contribution of x_t to u_t. The smaller the value of ρ^2, the larger the contribution. The derivation of the asymptotic distribution for our proposed statistic requires the following assumptions.

Assumption 2.1. For some $s>r>2$, we assume (a) (x_t,ε_t) is a covariance stationary and strong mixing process with mixing coefficients α_m, satisfying $\sum_{m=1}^{\infty}\alpha_m^{1/r-1/s}<\infty$; (b) $\sup_t \mathrm{E}(|x_t|^s+|\varepsilon_t|^s)<\infty$; (c) ε_t is a zero mean white noise process; (d) $\mathrm{E}(x_{t-k}\varepsilon_t) = 0$ for $-q_1\leq k\leq q_2$; and (e) $\alpha(L)\neq 0$ for all $|L|\leq 1$.

Assumptions 2.1 (a) and (b) are dependence and moment conditions, respectively, which are commonly used in the literature, including Hansen (1995). Assumptions 2.1 (c) and (d) imply that the regression error ε_t is orthogonal to the lagged differences of the dependent variable $(\Delta y_{t-1},\ldots,\Delta y_{t-p})$ and the leads and lags of the covariates $(x_{t+q_1},\ldots,x_{t-q_2})$, which is required to consistently estimate the parameters and construct our test in the next section. This requirement can be achieved by suitably increasing the orders of p,