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Conjectural variations, symmetric equilibria and economic policy

Ludovic A. Julien *

EconomiX, Université Paris Ouest-Nanterre La Défense, Bureau K116, 200 avenue de la République, 92001 Nanterre Cédex, France

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ABSTRACT

In this paper, we consider conjectural variations in a simple static general equilibrium model under oligopolistic competition. The modeling of conjectures captures the role played by beliefs in a micro-founded model. So, the economy may have three kinds of symmetric general equilibria. Furthermore, these equilibria can be Pareto-ranked by the conjectural variation parameter. Finally, we consider the implementation of a tax on the strategic behaviors in case of balanced-budget rule. The comparative statics illustrates the idea according to which the effectiveness of the multiplier mechanism to mitigate the market distortions depends on the symmetric equilibrium considered. Therefore, the effect of the tax on the prices and economic activity depends on the degree of market power which is conjectured by the agents.

1. Introduction

Some economists put forward the role of conjectures in strategic interactions (Dutt and Sen, 1995; Friedman and Mezzetti, 2002). The conjectural approach takes into account the perceptions by individuals of their market environment, and intends to study price formation without an auctioneer by attempting a general equilibrium analysis of imperfect competition (Gale, 1978; Hahn, 1977). These conjectures illustrate the way a firm belonging to a given industry anticipates the reactions of its direct rivals when it decides to increase its supply of a unit on the market (Bowley, 1924). The role played by (consistent) conjectures has mainly been developed in the context of production economies under partial equilibrium analysis (Bresnahan, 1981; Dixit, 1986; Figuières et al., 2004; Perry, 1982).

In this paper, we consider conjectural variations in a general equilibrium model with imperfect competition in the spirit of Hart (1982), Heller (1986), Jones and Manuelli (1992) or Roberts (1987). In order to simplify, we only focus on strategic interactions on the output markets and do not develop the labor market analysis.[1] More precisely, we propose to generalize the oligopoly-Nash model proposed by Cooper (1999).[2] We henceforth consider a large but finite number of goods, and put forward the role played by conjectural variations. Conjectural variations have already been introduced in a one sector imperfect competition model with wage-bargaining in the labor market in order to capture their influence on the markup (Dutt and Sen, 1995). Nevertheless, the preceding approach includes three shortcomings. First, it does not feature the role played by conjectural variations in the allocation of resources, while we model their influence on the equilibrium outcome. Second, the market demand addressed to each producer is exogenous; it is here derived from explicit optimizing behaviors. Third, it neglects the interactions between markets, whereas the present model generalizes the one good framework and emphasizes the strategic interactions among many agents in multiple interrelated markets.

This paper thus captures the role played by conjectural variations on market power and equilibrium allocation. The model can be represented as a two-step game: first, the market-clearing prices are determined for given strategies; second, the equilibrium strategies are determined at these equilibrium prices.[3] Several kinds of equilibria can arise. We restrict the analysis to symmetric equilibria.[4] Furthermore, we study the relationships between welfare, economic policy and conjectural variations. Four kinds of results are obtained. Firstly, the equilibrium prices and level of activity decrease with the conjectural variations parameter. Secondly, the economy may have three symmetric general equilibria. Thirdly, these symmetric equilibria can be Pareto-ranked by the conjectural variations parameter. Hence, the level of welfare associated with the competitive equilibrium allocation is highest. Fourthly, it is shown, on the one hand, that a per unit tax levied on the strategic supplies has a positive effect on the market outcome and, on the other hand, that the competitive equilibrium tax rate is the lowest equilibrium tax rate.

The paper is organized as follows. In Section 1, we describe the basic model. Section 2 is devoted to the analysis of the symmetric equilibria. Section 3 deals with welfare and economic policy. In Section 4, we conclude.

* Tel.: +33 1 40977356; fax: +33 1 40975907.
E-mail address: ludovic.julien@u-paris10.fr.

[1] It could be introduced without modifying the main features of the model.

[2] See Cooper (1999), Section 1, Chapter 3. Introducing several sectors has two advantages. Firstly, it generalizes the two goods framework and puts forward the strategic interactions among many markets. Secondly, the coincidence between imperfect and perfect competition equilibria can be captured by extending the size of the economy, instead of replicating it.

[3] See notably the concept of oligopoly equilibrium developed by Gabszewicz (2002).

[4] The model does not feature various degrees of competition among markets.

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2. The model

Consider an L-sector economy with Ln agents indexed h, $h=1, \ldots, Ln$, with n agents per sector. All agents are identical within a sector. There are L consumption goods indexed ℓ, $\ell=1, \ldots, L$, and a nonproduced good for which all agents have an endowment $\overline{m}_h$, $\forall h$. The price of this good is equal to 1, while the prices of good ℓ is denoted p_ℓ, $\forall \ell$. Each good ℓ, $\ell=1, \ldots, L$, is produced in quantities $y_{h\ell}$ according to a simple constant returns to scale technology, so total costs are a linear function of production. We assume that any agent does not consume the good he produces. As in Roberts (1987) and Weitzman (1982), this feature captures the decentralization of economic activities: the specialization in production and the generalization in consumption. Moreover, agents hold real balances m_h. The preferences of agent h who produces good ℓ are specified by the following utility function:

$$U_h = \prod_{k \neq \ell} \left(\frac{c_{hk}}{\alpha_k}\right)^{\alpha_k} \left(\frac{m_h}{1-\sum_{k \neq \ell}\alpha_k}\right)^{1-\sum_{k \neq \ell}\alpha_k} - \beta_\ell y_{h\ell}, \forall h \text{ for } k \neq \ell, \tag{1}$$

where α_k is the share of income used for consumption of good k, with $\alpha_k \in (0, 1)$ and $\Sigma_k \alpha_k = 1$, which also measures the strength of the demand linkage across all sectors. Additionally $\beta_\ell \in (0, 1)$, $\forall \ell$ is the marginal disutility of production for good ℓ.

The strategy set of oligopolist h who produces good ℓ may be defined as:

$$S_{h\ell} = \left\{ y_{h\ell} \in IR_+ \,|\, 0 \leq y_{h\ell} \leq \frac{1}{n}\frac{\Xi}{p_\ell} \right\}, \tag{2}$$

where $\frac{\Xi}{p_\ell}$ is the market demand function for good ℓ rationally perceived, and $\Xi \equiv \alpha_\ell \Sigma_{k \neq \ell} \Sigma_h \Omega_h(p_k, y_{hk})$ represents total expenditure of the agents who do not produce but consume good ℓ (see thereafter for a derivation).

Finally, it is considered that each agent forms conjectural variations. These conjectures denoted $\nu_{h\ell}$, $\ell=1, \ldots, L$, characterize the beliefs of every agent as for the reactions of his direct rivals when he decides to increase his supply of good ℓ of one unit (Bowley, 1924; Perry, 1982). Following Perry (1982), Dutt and Sen (1995), we consider only constant conjectures, so that the beliefs of every agent are independent from supplies of their rivals and from the number of agents. Other specifications are conceivable (Figuières et al., 2004; Kamien and Schwartz, 1983). The conjectural variations are thus defined as follows:

$$\frac{\partial \sum_{-h} y_{-h\ell}}{\partial y_{h\ell}} = \nu_{h\ell}, \ \forall h \text{ with } \nu_{h\ell} \in [-1, n-1], \ell = 1, \ldots, L. \tag{3}$$

It is also assumed that the conjectures of oligopolists belonging to the same industry are identical, i.e. $\nu_{h\ell} = \nu_\ell$, $\forall h$, $\ell=1, \ldots, L$. It means that only homogeneous conjectures prevail. In order to simplify, we consider locally consistent conjectural variations.[5]

Each of the n oligopolists who produces good ℓ selects first as a consumer his demands for all goods k, with $k \neq \ell$, and his demand for the nonproduced good. After, he determines as a producer his strategic supply $y_{h\ell}$. The program of consumer h may be written:

$$\underset{\left(c_h \in IR_+^{L-1}, m_h\right)}{Max} U_h(c_h, m_h, y_{h\ell}) \text{ s.t.} \sum_{k \neq \ell} p_k c_{hk} + m_h \leq p_\ell y_{h\ell} + \overline{m}_h, \forall h \text{ for } \ell \neq k. \tag{4}$$

For a given strategy $y_{h\ell} \in S_{h\ell}$ and given prices, the nL demand functions are $c_{hk} = \alpha_k \frac{\Omega_h}{p_k}$, $\forall k \neq \ell$ and $m_h = (1-\Sigma_k \alpha_k)\Omega_h$, where $\Omega_h \equiv p_\ell y_{h\ell} + \overline{m}_h$.

Each oligopolist then maximizes his indirect utility function in order to determine his strategic supply $y_{h\ell}$, taking as given the supply of his rivals, i.e. $\Sigma_{-h} y_{-h\ell}$, the price of the other goods p_k, $\forall k \neq \ell$ and the income Ω_{-h} of the $(nL-1)$ other agents. The program of producer h may be written:

$$\underset{\{y_{h\ell} \in S_{h\ell}\}}{\text{Arg max}} \prod_k p_k^{-\alpha_k} \left[p_\ell \left(y_{h\ell} + \sum_{-h} y_{-h\ell}\right) y_{h\ell} + \overline{m}_h \right] - \beta_\ell y_{h\ell}. \tag{5}$$

This leads to the n first-order conditions, where marginal revenue balances marginal cost:

$$\prod_k p_k^{-\alpha_k} \left[p_\ell + \left(\frac{\partial p_\ell}{\partial y_{h\ell}} + \frac{\partial p_\ell}{\partial \sum_{-h} y_{-h\ell}} \frac{\partial \sum_{-h} y_{-h\ell}}{\partial y_{h\ell}} \right) y_{h\ell} \right] = \beta_\ell, \ \forall h \text{ for } \ell \neq k. \tag{6}$$

These n optimality conditions reflect the fact that any oligopolist takes into account the reactions of his rivals through the term $\frac{\partial p_\ell}{\partial \sum_{-h} y_{-h\ell}} \frac{\partial \sum_{-h} y_{-h\ell}}{\partial y_{h\ell}}$. They also put into perspective sectoral and inter-sectoral strategic interactions and involve market equilibrium under consistent conjectures. The other $(L-1)n$ optimality conditions are similarly defined.

3. Symmetric general equilibria

A symmetric conjectural general equilibrium (SCGE) is a price level $\tilde{p}$, with $\tilde{p}_\ell = \tilde{p}$, $\forall \ell$, a level of production per firm $\tilde{y}_h$, with $\tilde{y}_{h\ell} = \tilde{y}_h$, $\forall h$, $\forall \ell$, a consumption vector $(\tilde{c}_h, \tilde{m}_h) \in IR_+^{L-1} \times IR_+$, $\forall h$ and a vector of conjectural variations $(\nu_1, \ldots, \nu_\ell, \ldots, \nu_L)$ such that the following three conditions hold: (i) $\nu_\ell = \nu$, $\forall \ell$, (ii) all markets simultaneously clear, and (iii) given $\tilde{p}$ and ν, each oligopolist optimizes at $\tilde{y}_h \in S_{h\ell}$ and at $(\tilde{c}_h, \tilde{m}_h) \in IR_+^{L-1} \times IR_+$.

The equilibrium concept is captured within a two-step game with complete but imperfect information.[6] First, each agent determines his best supply strategy taking as given the equilibrium price system (the market-clearing conditions) and the strategies of all other oligopolists. Second, the equilibrium prices which clear all markets are determined. The game is solved by backward-induction, so the price system, which clears all markets, is firstly determined, and after oligopolists interact in quantity spaces in order to determine their equilibrium strategies. Thus, the equilibrium prices is determined for given strategies and the equilibrium level of activity results from strategic interactions between reaction functions within quantity spaces.

Let us now compute the SCGE. Within each sector, each oligopolist rationally expects the equilibrium price. As it stands in Shapley and Shubik (1977), the market-clearing condition for good ℓ rationally expected by oligopolists is $p_\ell = \Xi / y_\ell$, $\forall \ell$,[7] and $y_\ell = \Sigma_h y_{h\ell}$. From Eq. (3), Eq. (6) becomes:

$$p_\ell \prod_k p_k^{-\alpha_k} \left[1 - \frac{(1+\nu_\ell)}{n} \right] = \beta_\ell, \text{ for } \ell \neq k, \tag{7}$$

where $1 - \frac{(1+\nu_\ell)}{n}$ is the markup,[8] which depends on ν_ℓ. At a symmetric conjectural general equilibrium, one obviously has: $\alpha_k = \alpha$, $\forall k$,

[5] The consistency of the individual beliefs relative to the reactions of the direct rivals concerns both the strategies (the individual supplies) and the reaction functions (Bresnahan, 1981; Perry, 1982). Therefore, the slope of the reaction function of the industry must coincide at an equilibrium point with the expected reaction that defines this conjecture.

[6] On the concept of Cournot–Walras equilibrium see notably Busetto et al. (2008).

[7] The market-clearing conditions involve absolute prices.

[8] The term $-1/n$ represents the inverse of the price elasticity of demand evaluated at the equilibrium (-1) times the market share of oligopolist h, i.e. $1/n$.