



The calculation of capital requirement using Extreme Value Theory

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ABSTRACT

The Basel Committee has suggested some formulas for calculating capital requirement using the Advanced Internal Ratings-Based Approach. However, these formulas were derived under the assumption of a normal distribution. Thus, the capital requirement estimated by the Basel formula may be incorrect when the asset distributions are not normal. Using an analysis of qualifying revolving retail exposures as an example, this paper introduces a formula based on the Extreme Value Theory to calculate the capital requirement. This formula is more general and accurate than its predecessors, because it can be used with any type of distribution. Numerical examples are provided to demonstrate that the capital requirement estimated by the Basel formula is less than by our formula when the asset distribution has a heavy tail, and more than by our formula when the distribution has a short tail. Our formula is also more sensitive to risk than competing models in the context of the recent financial crisis.

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1. Introduction

Methods for managing credit risk have undergone dramatic developments in the last 20 years because of the worldwide increase in the number of bankruptcies and the dramatic growth in the use of off-balance-sheet instruments to hedge the inherent risk of default (McKinsey, 1993; Altman and Saunders, 1998). According to the theory of credit management, a bank needs to maintain a rational capital adequacy for reducing its bankrupt probability. To address this need, the Basel Committee of Banking Supervision suggested a common framework (i.e. the Basel Capital Accord) for calculating the capital adequacy. Because the Basel Capital Accord is suggested to be implemented worldwide, research on the best way to calculate capital requirement has recently become popular among both academic scholars and financial institutions.

The recent version of the Basel Capital Accord is the “International Convergence of Capital Measurement and Capital Standards: A Revised Framework” (hereafter referred to as the RF). The RF suggests three methods for calculating the capital requirement of bank’s credit risk: the Standardized Approach, the Foundation Internal Ratings-Based Approach, and the Advanced Internal Ratings-Based Approach (hereafter referred to as the Advanced IRB Approach). In the studies related to Basel Capital Accord, the issues for accurately estimating the default probability and calculating the capital requirements have been widely discussed (Carey and Hrycay, 2001; Altman et al., 2002; Gordy, 2000; Altman, Resti and Sironi, 2003; Perli and Nayda, 2004).

Johnston (2009) derived a modified model for calculating the capital requirement that takes account of a bank’s earnings. Repullwo and Suarez (2004) proposed a net interest income correction model for IRB Approach. Gatzert and Schmeiser (2008) extended the Basel formulas to address the issues of bank’s equity investment. Most researchers calculate the credit exposure based on parameterized credit risk models. Grane and Veiga (2008) have argued that the assumption of a normal distribution is improper when estimating capital requirement. Brooks, Clareb and Persanda (2000) demonstrated that the use of GARCH-type models for the calculation of minimum capital risk requirements may lead to the production of inaccurate and therefore inefficient capital requirements. These authors give a simple modification to the model to improve the accuracy of minimum capital risk requirements estimates in both back- and out-of-sample tests. Some researchers have used nonparametric statistics to generate the credit-loss distributions needed to calculate the credit risk (Jacobson et al., 2005).

The RF pays particular attention to retail credit¹ because of the supposedly smaller exposure to systemic risk that it entails (Jacobson et al., 2005). In the RF, retail credit exposures are divided into three primary categories: (1) exposures secured by residential mortgages, (2) qualifying revolving retail exposures (hereafter referred to as QRRE), and (3) other non-mortgage exposures, also known as “other retail” (RF paragraphs 233). This paper uses the QRRE as an example to introduce a new formula for calculating the capital requirement. QRRE consists of unsecured revolving credits that exhibit appropriate loss characteristics. For example, the credit card is a kind of QRRE. A

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¹ The retail credit business in commercial banks involves consumer credit, residential mortgages, automobile loans, credit cards, and small business loans (Allen et al., 2004).

formula for calculating the capital requirement of QRRE is provided by the Advanced IRB approach. In so doing, bank can use a suitable distribution for estimating the probability default, because the distributions of financial data usually show heavy tails. The estimated default probability then is entered into the suggested formula for calculating the capital requirement.

The RF takes into consideration the above-mentioned problems of skewed distributions in calculating the default probability. However, the calculation of the capital requirement may still be inaccurate because the formula for calculating the critical value of the default probability at a given confidence level is still derived on the assumption that the asset probability is normally distributed (Perli and Nayda, 2004). The purpose of this paper is to derive a formula for accurately calculating the capital requirement under a more general framework.

The Extreme Value Theory has been widely applied in the finance and insurance literature to analyze data distributions characterized by extreme departure from normality with respect to skewness. Specific applications include portfolios, operational losses, catastrophic insurance claims, and credit losses (Embrechts et al., 1997; Fernández, 2003a,b; Coles, 2001; Reiss and Thomas, 2001; McNeil and Frey, 2000). To accurately calculate the capital requirement, we use the Extreme Value Theory to derive a formula for accurately calculating the critical value of default probability at a given confidence level. Our formula is more general and estimates capital requirement more accurately than previous models because it can be applied to any type of distribution (e.g., the distribution with heavy tails). We provide quantitative examples comparing our formula with that suggested by the RF. The results show that the RF formula can underestimate capital requirements when the sample distribution has a heavy tail and overestimate it when it has a short tail.

Since 2007, the sub-prime mortgage crisis has kept US financial markets in turmoil and eventually created turbulence in financial markets worldwide. During this period, many famous banks and insurance companies, such as Lehman Brothers, Merrill Lynch, Fannie Mae, and Freddie Mac, have been overleveraged or burdened by debt, resulting in bankruptcies or takeovers. Because maintaining adequate capital is how banks avoid default, any formula that adequately estimates a bank's capital requirement must accurately reflect the huge turbulence in the financial markets. We also use a numerical example to show how our formula is more sensitive to risk when economic situations change.

The remainder of this paper is organized as follows. In Section 2 we present the formula for calculating the capital requirement of QRRE as described in the New Basel Accord. We also present the formula for calculating the default probability based on the Extreme Value Theory (the RF approach). In Section 3, we use the Extreme Value Theory to derive a new formula for calculating the critical default probability at a given confidence level when the default rate distribution is non-normal with respect to skewness. Section 4 includes quantitative examples comparing the formulas. Finally, in Section 5, we summarize our main findings and offer suggestions for future research.

2. Formulas for calculating default probabilities using the Extreme Value Theory

The standard procedure for determining the QRRE can be described in terms of the following equations (defined in RF paragraph 329):

$$CR = VaR_{\alpha}(PD) \times LGD, \quad (1)$$

$$VaR_{\alpha}(PD) = PD_{\alpha} - PD, \quad (2)$$

$$PD_{\alpha} = \Phi\left(\frac{\sqrt{\rho}\Phi^{-1}(\alpha) + \Phi^{-1}(PD)}{\sqrt{1-\rho}}\right), \text{ and} \quad (3)$$

$$\text{Risk-weighted assets} = CR \times 12.5 \times EAD, \quad (4)$$

where

CR	is the capital requirements,
LGD	is the loss given default,
EAD	is the exposure at default,
α	is the confidence level, which is suggested to be 99.9% in the RF,
PD	is the probability of default, which can be estimated with a proper method,
$VaR_{\alpha}(PD)$	is the Value-at-Risk of the default probability under confidence level α ,
PD_{α}	is the critical value of PD under α confidence level,
ρ	is the correlation of consumers' assets, suggested as $\rho = 0.04$ in RF,
$\Phi(\cdot)$	is the cumulative normal distribution, and
$\Phi^{-1}(\cdot)$	is the inverse cumulative normal distribution function.

RF suggests that a specific bank can use its own estimation methods to determine LGD, EAD and PD for calculating its capital requirement. In this section, we derive a formula for calculating the default probability, taking into account the skewness problems, based on the Extreme Value Theory.

To begin, let $V_i(T)$ represent the standardized asset value of debtor i at time T , and let k be a given critical asset value of bankruptcy. According to the traditional definition of bankruptcy, default occurs when $V_i(T) < k$. Therefore, the probability of default for an individual consumer can be described as follows:

$$PD_i = \Pr(V_i(T) < k), \quad i = 1, 2, \dots, N, \quad (5)$$

where N is the number of consumers.

Let a debtor's asset value be driven by a single common factor Y , such as a macroeconomic variable related to the business cycle (e.g. national income, consumer price index, stock index). We can describe the standardized asset value as follows (Perli and Nayda, 2004):

$$V_i(T) = \sqrt{\rho}Y + \sqrt{1-\rho}\varepsilon_i, \quad (6)$$

where ε_i is an individual variable. Assume that Y and ε_i are independent. Let $F(\cdot)$ be the cumulative conditional probability distribution of ε_i ; $G(\cdot)$ be the cumulative conditional probability distribution of Y ; and $Q(\cdot)$ be the cumulative conditional probability distribution of $V_i(T)$. According to Eq. (3), $Q(\cdot)$ is a combination of the distributions of Y and ε_i . In the study of Perli and Nayda (2004), $F(\cdot)$, $G(\cdot)$, and $Q(\cdot)$ were assumed to be standard normal distributions for deriving the formula suggested in the RF (hereafter referred to as the RF formula). Here we relax this assumption and support a more general formula for estimating the default probability.

The default probability contingent on the realization of the common factor ($Y=y$) can be described as follows:

$$PD \equiv PD(y) = \Pr(V_i(T) < k | Y = y) = \Pr\left(\varepsilon_i < \frac{k - \sqrt{\rho}y}{\sqrt{1-\rho}}\right) \equiv F(s), \quad (7)$$

$$\text{where } s = \frac{k - \sqrt{\rho}y}{\sqrt{1-\rho}}$$

This default probability, $F(s)$, can be estimated from the Extreme Value Theory. In practice, there are two methods for analyzing

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