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Estimates for the optimal control policy in the presence of regulations and heavy tails

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ABSTRACT

We consider a classical heavy tailed risk model, included in a regulation mechanism. The regulator exercises a minimal cash requirement level and penalties for violating it to regulate the insurance firm. The problem of the insurance firm is to establish an investment and risk exposure policy as well as a barrier dividend strategy, which is a function of the strategy used by the regulator. For regularly varying tailed claim size distributions, we find the asymptotics of the stationary distribution of the risk model and derive fundamental asymptotic results of the insurance firm's problem. In the special case of Pareto claim size distributions, the asymptotic optimal control policy is found in closed form, as well as numerical results.
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1. Introduction

There are numerous studies on the reasons for and the impact of regulating insurance firms. The main aim of regulations is to help regulators identify weak insurers and take prompt corrective action to intervene when capital falls below specified levels. The main regulations in this regard are the solvency regulations, which involve both the solvency and the capital adequacy requirements.

With regulations, the regulator's cost function is a quantitative description of the protection given to the insurance holders from insurance firm's failures, and the efficiency and lack of competition due to government interventions at the same time. To our knowledge, few papers of regulations have focused on the regulator's cost structure but on the effect of insurance firms' profit, except Tapiero et al. (1983). By choosing the investment financing, and its link with optimal risk exposure, insurance holders or regulators would use ruin probability of insurance firms as an objective function. The calculation of ruin probabilities in the classical risk model by now is well understood (see Schimidli (2007) as a survey). Many recent papers have also studied the same problem for the case of heavy tails (see Konstantinides et al., 2002; Gaier and Grandits, 2002; Hipp and Plum, 2000; Tang, 2005; Schimidli, 2002, 2005), while shareholders would prefer other objective functions such as expected discounted dividends (see David and Drekic, 2006; Schimidli, 2007).

In this paper we continue the work of the cited papers, and extend the study of Tapiero et al. (1983) to proportional reinsurance and the claim sizes with *heavy*, especially regularly varying tails. As in Tapiero et al. (1983), we assume a barrier investment strategy, distinguishing between short term and long term investments. The problem of the regulator is to regulate the insurance firm by establishing a minimal cash requirement level and penalties for violating it. Meanwhile, the insurance firm maximizes the long run average profit function by choosing a control policy. That is, it has to find out an investment and risk exposure policy as well as a barrier dividend strategy, which is a function of the strategy used by the regulator. In this case the main work one has to do, is to find the asymptotics of a modified convolution integral in the integro-differential equation which the stationary distribution satisfies.

The paper is organized as follows. We construct a heavy tailed claim model in Section 2 and provide fundamental asymptotic results in Section 3. In Section 4, we introduce the regulator's problem and derive its optimal policy. Furthermore, an explicit asymptotic and closed form expression for the long run average profit function of the insurance firm is found in the special case of Pareto claim size. We also derive the closed form of the asymptotic optimal control policy of the insurance firm and the regulator in Section 5, respectively. Finally, numerical results of the obtained results are illustrated.

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2. The model formulation

We consider an insurance firm with insurance risk modeled by a classical heavy tailed risk process

$$X(t) = x_0 + ct - \sum_{i=1}^{N(t)} S_i, \quad t \geq 0, \tag{1}$$

in which $\sum_{i=1}^{N(t)} S_i$ is a compound Poisson process with intensity λ and claim size distribution F, and x_0 is the initial surplus. We assume the premium rate c has a positive security loading factor θ such that $c = (1+\theta)\lambda\xi$, where ξ is the finite mean claim size.

We denote the equilibrium distribution (e.d.f.) of F by

$$G(y) = \frac{1}{\xi}\int_0^y \overline{F}(z)dz, \qquad y \geq 0, \tag{2}$$

where $\overline{F}(z) = 1 - F(z)$. By definition, an e.d.f. G supported on $[0,\infty)$ is called regularly varying with index ρ, if it holds that

$$\lim_{y\to\infty} \frac{\overline{G}(xy)}{\overline{G}(y)} = x^{\rho}, \qquad x > 0. \tag{3}$$

If $\rho = 0$, we say that G is slowly varying.

Under regulations, let K be the regulation barrier and R be the penalty cost rate imposed by the regulator. The insurance firm has to establish an investment, proportional reinsurance and dividend strategy. The detailed policy is four fold. That is:

1. The proportional reinsurance strategy. Reinsurance allows the insurance firm transferring a proportion $1-a$ of each claim paid by another insurance firm, and paying reinsurance premium with the same security loading factor.
2. The investment strategy. All incoming premiums above K are directly transferred into short term investments with a return rate r_1. If the amount of short term investments reaches a certain level S, all additional premiums are converted into long term investments with a return rate $r_2 \geq r_1$.
3. The barrier dividend strategy. All incoming premiums are distributed as a dividend to the shareholders by the insurance firm, when short term investments reach S and long term investments reach another control level L.
4. The capital converting strategy. If the cash level jumps below K, short term investments should be converted into cash to cover claims. While the amount of short term investments becomes zero, a penalty cost of monetary units below K at rate R is incurred for violating the regulation barrier K. And the insurance firm can borrow money with an interest rate r to cover claims if the cash level jumps below zero. For brevity, we assume the conversion time is negligible, no transaction cost occurs associated with the above conversions, and we restrict short term investments from being converted into cash.

Let $Y(t)$ be the amount of cash plus short term investments at time t, which is defined on $(-\infty, K+S]$. Furthermore, denote $Z(t)$ as the amount of long term investments on $[0, L]$, where L is an absorbing state. For Pareto claims, Fig. 1 illustrates the dynamics of $Y(t)$, $Z(t)$, and the amount of distributed dividends denoted by $D(t)$.

Similar to Tapiero et al. (1983), the optimal control policy is to maximize the long run average profit per unit time. Assuming that the administrative and commission costs are independent of the control policy, the costs of the insurance firm are the penalty cost and the borrowing cost. Then the firm's profit includes earned premiums plus income from short term and long term investments less reinsured claims and costs. Let $\psi(a,S,L|K,R)$ denote the long run average profit function when (a,S,L) control policy is adopted given the regulation barrier K and the penalty cost rate R.

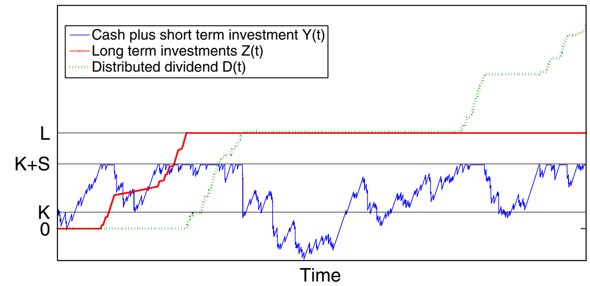

Fig. 1. The modified surplus process with Pareto claims.

It is obvious that the expected earned premiums less reinsured claim per unit time is $a\lambda\theta\xi$. Then the firm's objective function is given by

$$\psi(a,S,L|K,R) = a\lambda\theta\xi + \lim_{t\to\infty}\frac{1}{t}E\left\{\int_0^t r_2 Z(t)dt + \int_0^t r_1(Y(t)-K)I_{\{Y(t)\geq K\}}dt - \int_0^t R(K-Y(t))I_{\{Y(t)<K\}}dt + \int_0^t rY(t)I_{\{Y(t)<0\}}dt\right\}, \tag{4}$$

where $I_{\{E\}}$ is the indicator function of the event E. We will derive an explicit expression of $\psi(a,S,L|K,R)$ in the remainder.

We apply a renewal argument as in Tapiero et al. (1983). Without loss of generality, let

$$Y(0) = K + S, \quad Z(0) = L, \quad T_0 = \inf\{t > 0; Y(t) < K + S\}, \tag{5}$$

where T_0 denotes the first time when the stochastic process Y falls below the initial state $K+S$.

Similarly, define a stopping time T

$$T = \inf\{t > T_0; Y(t) = K + S\}, \tag{6}$$

which indicates the time when Y returns to the state $K+S$ after time T_0. Since the bivariate process (Y,Z) regenerates itself, we refer to the time period $[0,T)$ as a cycle. By the renewal argument, the objective function Eq. (4) can be expressed by

$$\psi(a,S,L|K,R) = a\lambda\theta\xi + r_2 L + \frac{1}{E[T]}E\left[\int_0^T f(Y(t))dt\right], \tag{7}$$

where

$$f(y) = \begin{cases} r_1(y-K), & \text{if} \quad K \leq y \leq K+S, \\ -R(K-y), & \text{if} \quad 0 \leq y < K, \\ -R(K-y) + ry, & \text{if} \quad y < 0. \end{cases} \tag{8}$$

Now, we have to determine the stationary distribution of the process Y, and define for $y>0$

$$W(y,a,t) = P\{Y(t) \geq K + S - y\}. \tag{9}$$

Let $W(y,a) = \lim_{t\to\infty} W(y,a,t)$ with the condition $\lim_{t\to\infty}\partial W(y,a,t)/\partial t = 0$. Hence, $W(y,a)$ satisfies the following integro-differential equation

$$\frac{\partial}{\partial y}W(y,a) - \frac{\lambda}{ac}\int_0^y \overline{F}\left(\frac{y-x}{a}\right)\frac{\partial W(x,a)}{\partial x}dx - \frac{\lambda\theta}{ac(1+\theta)}\overline{F}\left(\frac{y}{a}\right) = 0, \tag{10}$$

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