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Modeling input–output impacts with substitutions in the household sector: A numerical example

Peter Gordon [a,1], JiYoung Park [b,*], Harry W. Richardson [c,2]

[a] Ralph & Goldy Lewis Hall 321, School of Policy, Planning, and Development, University of Southern California, Los Angeles, CA 90089-0626, United States
[b] 201F Hayes Hall, Department of Urban and Regional Planning, University at Buffalo, The State University of New York, Buffalo, NY 14214-3087, United States
[c] Ralph & Goldy Lewis Hall 212, School of Policy, Planning, and Development, University of Southern California, Los Angeles, CA 90089-0626, United States

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A B S T R A C T

Although there have been many elaborations of the basic input–output approach, including multi-regional models, dynamic models, models with variable coefficients, supply-side models, etc., these approaches all have the same limitation. The fixed-coefficients production function assumptions ignore substitutions in response to price changes that can be expected to accompany most shocks— skipping over the heart and soul of market economics. This research note suggests a simple approach to estimating new technical coefficients matrices after a shock so that the consequences of short-term substitution effects can be studied. Given a reduction in income (as reflected in the value added row), households are likely to make substitutions, reducing their final demand by less than the application of base-year I–O coefficients would indicate. But if ex post changed income and consumption can be observed, the application of *RAS* procedures can generate an appropriately modified **A** matrix. The resulting set of interdependent substitutions that occurred can be identified. Due to some well known limits in applying the traditional RAS approach, we reformatted it and suggest a new economic model that can link coefficient adjustments to degrees of a priori substitutability and complementarity. Based on this resolution, we look forward to detailed studies of specific coefficients and how they evolve over the short term.

1. Issues

The application of input–output (I–O) models to economic impact studies is widespread and long established. A Google-scholar search recently found 677,000 I–O hits and 87,500 when the phrase is qualified with the words “economic impact”.

In most cases, the applications have changed little in over 50 years. The typical “what if” scenario is: If the vector of final demands (multiplicand) is perturbed in a particular way, what happens to the vector of total outputs in light of the Leontief inverse (multiplier)?

Of course, there are many elaborations of the basic approach, including multi-regional models, dynamic models, models with variable coefficients, supply-side models, etc. (Hewings, 1985; Miller and Blair, 1985).

All these approaches have the same limitation. The fixed-coefficients production function assumptions ignore substitutions in response to price changes — skipping over the heart and soul of market economics. The standard defense against this criticism is that the analysis is short-term and applicable to the interim, before agents have been able to discover the relevant substitution options.

The point of this note is simple. If the analyst can observe simultaneous changes in the final demand column and the value added row, he can use this information to study the many simultaneous and interdependent economic adjustments reflected in an impacted technical coefficients matrix; there are new multiplier values that reflect large numbers of real world substitutions, for example, made by the household sector. Facing a decline in final demand, firms may cut their wage bills asymmetrically; this may result in changes in household spending behavior that changes the multiplier impact.

2. Numerical example

A numerical example can corroborate our intuition. Given a reduction in income (as reflected in the value added row), households will attempt to make substitutions, reducing the utility of their consumption by less than the application of the base-year I–O coefficients would indicate. If ex post changes in income and consumption can be observed, an application of *RAS* procedures can generate the appropriately modified **A** matrix.[3]

* Corresponding author. Tel.: +1 716 829 3485.
E-mail addresses: pgordon@usc.edu (P. Gordon), jp292@buffalo.edu (J. Park), hrichard@usc.edu (H.W. Richardson).
[1] Tel.: +1 213 740 1467.
[2] Tel.: +1 213 740 3954.

[3] Traditional or alternative *RAS* methods assume that the target year's intermediate output and input vectors and total output vector are known in order to estimate the target year's coefficients (Jackson and Murray, 2004). Although we pointed to our approach as an application of RAS procedures, we do not require that the three vectors for the target year are known; our approach only depends on the impacted intermediate output (final demand) and intermediate input (value added) sectors. The impacted sectors are a subset of (or same as) the intermediate output and the intermediate input vectors for the target period.

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Table 1
Example of 2×2 hypothetical shipments matrix.

	x_1	x_2	F	X^O
x_1	150	500	350	1000
x_2	200	100	1700	2000
V	650	1400	1100	3150
$\mathbf{X}^I$	1000	2000	3150	6150

Source: Miller and Blair (1985), p.15.
Where x_i identifies industry sector i.
F = total intermediate output (final demand) column vector.
V = total intermediate input (value added) row vector.
$\mathbf{X}^o$ = total output column vector.
$\mathbf{X}^I$ = total input (outlay) row vector.

Consider the well known and highly simplified 2×2 example (Table 1), built upon Miller and Blair's (1985, p. 15). First, denote the 2×2 interindustry flow matrix as $\mathbf{Z}\left(=\begin{pmatrix}150 & 500\\ 200 & 100\end{pmatrix}\right)$. Given the 2×2 flow matrix, we use the open model. The matrix **A** is calculated as

$$\mathbf{A}=\mathbf{Z}\left(\hat{\mathbf{X}}^I\right)^{-1}=\begin{pmatrix}0.15 & 0.25\\ 0.20 & 0.05\end{pmatrix} \tag{1}$$

where, the hat on the vector $\mathbf{X}^I$ indicates a diagonal matrix transformed from $\mathbf{X}^I$,

$$\hat{\mathbf{X}}^I=\hat{\mathbf{X}}^O=\begin{pmatrix}1000 & 0\\ 0 & 2000\end{pmatrix}, \text{and} \tag{2.1}$$

$$\left(\hat{\mathbf{X}}^I\right)^{-1}=\left(\hat{\mathbf{X}}^O\right)^{-1}=\begin{pmatrix}\frac{1}{1000} & 0\\ 0 & \frac{1}{2000}\end{pmatrix}. \tag{2.2}$$

Therefore, the Leontief inverse matrix is calculated as,

$$(\mathbf{I}-\mathbf{A})^{-1}=\begin{pmatrix}1.254 & 0.330\\ 0.264 & 1.122\end{pmatrix}. \tag{3}$$

Based on $\mathbf{X}^o$, the matrix **B** is calculated as

$$\mathbf{B}=\left(\hat{\mathbf{X}}^O\right)^{-1}\mathbf{Z}=\begin{pmatrix}0.15 & 0.50\\ 0.10 & 0.05\end{pmatrix}. \tag{4}$$

The matrix **B** is the so-called Goshian inverse matrix as[4]

$$(\mathbf{I}-\mathbf{B})^{-1}=\begin{pmatrix}1.254 & 0.660\\ 0.132 & 1.122\end{pmatrix}. \tag{5}$$

As a simple example of a hypothetical impact analysis, we assume the observed exogenous losses occur in the value added vector as income decreases from an exogenous shock,

$$\Delta\mathbf{V}=(-100 \quad -300), \tag{6}$$

Total output losses resulting from the exogenous losses via the Goshian inverse matrix would be estimated as,

$$\left(\Delta\mathbf{X}^O\right)^T=\Delta\mathbf{V}(\mathbf{I}-\mathbf{B})^{-1}=(-165.0 \quad -402.6) \tag{7}$$

where the superscript T represents the transpose of the matrix.

Under the assumption that other intermediate input factors are not changed, reduced labor inputs due to a shock may decrease total outputs, increasing prices of the related sectors.[5] Also, consumption expenditure by households will be affected in response to their decreased income as well as any increased prices of products.

Consider decreased output first. A newly derived total output column vector ($\mathbf{NX}^o$) is calibrated as,

$$\mathbf{NX}^O=\mathbf{X}^O+\Delta\mathbf{X}^O=\begin{pmatrix}1000\\ 2000\end{pmatrix}+\begin{pmatrix}-165.0\\ -402.6\end{pmatrix}=\begin{pmatrix}835\\ 1597\end{pmatrix}. \tag{8}$$

The reduced final demand in next period (**NF**) resulting from the losses of income can be obtained from the following equation, using the proportion of the final demand to total output in the previous period.

$$\mathbf{NF}=\hat{\mathbf{P}}_{\mathbf{X}^O}\mathbf{NX}^O=\begin{pmatrix}0.35 & 0\\ 0 & 0.85\end{pmatrix}\begin{pmatrix}835\\ 1597\end{pmatrix}=\begin{pmatrix}292\\ 1358\end{pmatrix} \tag{9}$$

where the proportionate column vector,

$$\mathbf{P}_{\mathbf{X}^O}=\left(\hat{\mathbf{X}}^O\right)^{-1}\mathbf{F}=\begin{pmatrix}\frac{1}{1000} & 0\\ 0 & \frac{1}{2000}\end{pmatrix}\begin{pmatrix}350\\ 1700\end{pmatrix}=\begin{pmatrix}0.35\\ 0.85\end{pmatrix}. \tag{10}$$

From the result in Eq. (9), therefore, the final demand losses (Δ**NF**) between two periods are calculated as

$$\Delta\mathbf{NF}=\mathbf{NF}-\mathbf{F}=\begin{pmatrix}292\\ 1358\end{pmatrix}-\begin{pmatrix}350\\ 1700\end{pmatrix}=\begin{pmatrix}-58\\ -342\end{pmatrix}. \tag{11}$$

At the same time, we can observe the concurrent losses of final demand, Δ**F**. The losses of final demand are assumed to be smaller than those implied by the observed Δ**V** from Eqs. (6)–(11), the Δ**NF**, because households experiencing income losses will seek substitutions that mitigate negative consumption impacts.[6] The economic

[4] In the I–O world, two standard models have been developed: Leontief demand-driven and Goshian supply-driven models. While Leontief (1936) first generalized inter-industry relations throughout an economy responding to a change in final demands and generating new requirements for industrial inputs, Ghosh (1958) observed that a non-scarce resource in a centrally planned economy where all other resources are scarce is not induced via demand-driven effects but controlled via an allocation function. The latter is the so-called supply-driven model. If the losses are in final demand (intermediate output) factors, then the Leontief demand-driven model is applied. If the losses are in value added (intermediate input) factors, then the Goshian supply-driven model is used. Dietzenbacher's (1997) interpretation and application of the two models seemed novel until now.

[5] The Goshian supply-driven model can be viewed as a price model; it can capture price-effects as quantity changes (Dietzenbacher, 1997). However, it needs further discussion for the direction of price changes when labor inputs are decreased, e.g. due to a shock (Park, 2007).

[6] As an anonymous referee pointed out, the assumption that the Δ**F** is less than the Δ**NF** might not appear in the actual market, unless some correspondence with reality are provided. Because this study is the first trial to suggest our model, yet this assumption needs to be tested in a following study, based on real data., e.g. BEA (Bureau of Economic Analysis)'s national I–O data.