ELSEVIER

Sustainability of US public debt: Estimating smoothing spline regressions[☆]

Alfred Greiner [*], Göran Kauermann

Department of Business Administration and Economics, Bielefeld University,
P.O. Box 100131, 33501 Bielefeld, Germany

Abstract

In this paper we analyze how the primary surplus to GDP ratio in the US reacts to variations in the public debt-GDP ratio. In contrast to earlier studies we perform non-parametric and semi-parametric estimations. Our results show that the response of the primary surplus to GDP ratio is a positive nonlinear function of the public debt-GDP ratio. Further, our estimations demonstrate that the coefficient giving the response of the primary surplus-GDP ratio to a change in the public debt-GDP ratio declines over time when we assume a linear model with time dependent coefficients.
© 2006 Elsevier B.V. All rights reserved.

1. Introduction

Bohn (1998) presented a new sustainability test for public debt because, as he argues, conventional tests are restrictive since they conclude too soon that a given policy is unsustainable. As an alternative, Bohn suggests to investigate how the primary surplus reacts to increases in the public debt-GDP ratio. If the primary surplus to GDP ratio increases at least linearly with a rising public debt-GDP ratio, public debt is sustainable. Intuitively, the reasoning behind this test is obvious. If the government raises the primary surplus to GDP ratio as the debt to GDP ratio

increases it takes corrective actions which guarantee that the government remains solvent. Formally, this was proven by Bohn (1998) assuming a constant reaction coefficient. Canzonerie et al. (2001) generalized the proof by allowing for a time dependent reaction coefficient.

For the US, Bohn (1998) estimated OLS regressions and found that the primary surplus-GDP ratio is a positive function of the public debt-GDP ratio implying that US debt policy is sustainable. In addition, Bohn (1998) estimated parametric regression where the public debt-GDP ratio enters the equation to be estimated in a nonlinear way. He found evidence that the marginal effect of the public debt-GDP ratio on the primary surplus-GDP ratio rises with higher public debt-GDP ratios suggesting a nonlinear, convex relationship. The latter is also supported by Sarno (2001) who tested for nonlinearities and detected strong evidence for nonlinearities in the process generating the US debt-GDP time series.

However, neither of these authors apply non-parametric estimation techniques and they do not present an explicit estimate for the function giving the relation between the primary surplus-GDP ratio and the public debt-GDP ratio. In this paper we take the next step by exploring functional shapes of the effects. To do so, we take advantage of recent developments in statistics. About a decade ago Hastie and Tibshirani (1990) introduced Generalized Additive Models as a new flexible class of regression model. The theoretical achievements were accompanied by numerical developments which led to the success of the software packages S-PLUS and later on R available under http://www.r-project.org/ (see also Venables and Ripley, 2002). The main idea behind this class of models is, that the effect of an explanatory variable on some measurement of interest is not modelled as parametric (usually linear) function, but kept flexible by just postulating that the effect is smooth in the sense of differentiability. The functional shape is thereby to be estimated from the data, either using local, that is kernel based methods, or spline smoothing. Available software readily allows to fit such models. With the contributions by Wood (2000, 2001) also the disputable point of choosing the right amount of smoothing has been settled in practice. A motivating overview of the state of art in this field can be found in Ruppert, Wand and Carroll (2003). Additionally we pursue inferential aspects by testing functional estimates with the intention to find a parsimonious model. We pursue an F test extension in the line of Bowman and Azzalini (1997) where the distribution under H_0 is derived by bootstrapping.

In this paper we apply non-parametric estimation to US data. We are focussing the question whether the relation between the primary surplus to GDP ratio and the debt-GDP ratio is characterized by nonlinearities. In particular, we want to visualize the function governing the response of the primary surplus-GDP ratio to changes in the public debt-GDP ratio. To get insight into this question we pursue smoothing spline regression. Furthermore, we estimate a semi-parametric regression where we assume a linear relation between the primary surplus-GDP ratio and the public debt-GDP ratio. However, the coefficients associated with the predictor variables are assumed to be time varying. This leads to a varying coefficient model as introduced in Hastie and Tibshirani (1993).

The rest of the paper is organized as follows. In Section 2, we make some theoretical considerations and demonstrate that a given fiscal policy is sustainable if the primary surplus to GDP ratio is a positive linear function of the public debt-GDP ratio provided that the reaction coefficient is positive on average. In Section 3 we test for nonlinearities in the relationship between the primary surplus to GDP ratio and the debt-GDP ratio. Section 4 estimates a semi-parametric function where the exogenous variables enter the equation to be estimated in a linear way but the coefficients are allowed to be nonlinear functions of time. Section 5, finally, concludes.