Economics Letters 161 (2017) 52–55

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

The correct regularity condition and interpretation of asymmetry in EGARCH[☆]

Chia-Lin Chang[a,b,*], Michael McAleer[c,d,e,f,g]

[a] Department of Applied Economics, National Chung Hsing University, Taichung, Taiwan
[b] Department of Finance, National Chung Hsing University, Taichung, Taiwan
[c] Department of Quantitative Finance, National Tsing Hua University, Taiwan
[d] Discipline of Business Analytics, University of Sydney Business School, Australia
[e] Econometric Institute, Erasmus School of Economics, Erasmus University Rotterdam, The Netherlands
[f] Department of Quantitative Economics, Complutense University of Madrid, Spain
[g] Institute of Advanced Sciences, Yokohama National University, Japan

HIGHLIGHTS

- EGARCH is derived from a random coefficient complex nonlinear moving average stochastic process that leads to its specification.
- Asymmetry and leverage are discussed.
- The correct regularity condition for asymmetry in EGARCH is derived.
- The correct interpretation is provided.

ARTICLE INFO

Article history:
Received 6 July 2017
Received in revised form 13 September 2017
Accepted 14 September 2017
Available online 21 September 2017

JEL classification:
C22
C52
C58
G32

Keywords:
Conditional volatility models
Random coefficient complex nonlinear moving average process
EGARCH
Asymmetry
Leverage
Regularity condition

ABSTRACT

In the class of univariate conditional volatility models, the three most popular are the generalized autoregressive conditional heteroskedasticity (GARCH) model of Engle (1982) and Bollerslev (1986), the GJR (or threshold GARCH) model of Glosten et al. (1992), and the exponential GARCH (or EGARCH) model of Nelson (1990, 1991). For purposes of deriving the mathematical regularity properties, including invertibility that relates the standardized residuals to the returns shocks, to determine the likelihood function for estimation, and the statistical conditions to establish asymptotic properties, it is essential to understand the stochastic properties underlying the three univariate models. The random coefficient autoregressive process was used to obtain GARCH by Tsay (1987), an extension of which was used by McAleer (2014) to obtain GJR. A random coefficient complex nonlinear moving average process was used by McAleer and Hafner (2014) to obtain EGARCH. These models can be used to capture asymmetry, which denotes the different impacts on conditional volatility of positive and negative shocks of equal magnitude, and possibly also leverage, which is the negative correlation between returns shocks and subsequent shocks to volatility (see Black 1976). McAleer (2014) showed that asymmetry was possible for GJR, but not leverage. McAleer and Hafner (2014) showed that leverage was not possible for EGARCH. Surprisingly, the condition for asymmetry in EGARCH seem to have been ignored in the literature, or has concentrated on the incorrect parametric condition, with no clear explanation, and hence with associated unclear and misleading interpretations. The purpose of the paper is to derive the regularity condition for asymmetry in EGARCH, and to provide the correct interpretation. It is shown that, in practice, EGARCH always displays asymmetry, though not leverage.

"They're digging in the wrong place!"
Indiana Jones, Raiders of the Lost Ark

1. Introduction

In the class of univariate conditional volatility models, the three most popular are the generalized autoregressive conditional

[☆] The authors are most grateful to an Associate Editor and a referee for insightful comments and suggestions. For financial support, the first author wishes to thank the National Science Council, Ministry of Science and Technology (MOST), Taiwan, and the second author is grateful to the Australian Research Council and the National Science Council, Ministry of Science and Technology (MOST), Taiwan.
* Corresponding author at: Department of Applied Economics, National Chung Hsing University, Taichung, Taiwan.
E-mail address: changchialin@email.nchu.edu.tw (C.-L. Chang).

http://dx.doi.org/10.1016/j.econlet.2017.09.017

heteroskedasticity (GARCH) model of Engle (1982) and Bollerslev (1986), the GJR (or threshold GARCH) model of Glosten et al. (1992), and the exponential GARCH (or EGARCH) model of Nelson (1990, 1991). Each of these models is widely read, used in practice, and highly cited.

For purposes of deriving the mathematical regularity properties, including invertibility, to determine the likelihood function for estimation, and the statistical conditions to establish asymptotic properties, it is convenient to understand the stochastic properties underlying the three univariate models. The random coefficient autoregressive process was used to obtain GARCH by Tsay (1987), an extension of which was used by McAleer (2014) to obtain GJR. A random coefficient complex nonlinear moving average process was used by McAleer and Hafner (2014) to obtain EGARCH.

These models can be used to capture asymmetry, which denotes the different impacts on conditional volatility of positive and negative shocks of equal magnitude, and possibly also leverage, which is the negative correlation between returns shocks and subsequent shocks to volatility (see Black, 1976). McAleer (2014) showed that asymmetry was possible for GJR, but not leverage. McAleer and Hafner showed that leverage was not possible for EGARCH.

Surprisingly, the condition for asymmetry in EGARCH seems to have been ignored in the literature, or has concentrated on an incorrect condition, with no clear explanation, and hence with associated misleading interpretations. The purpose of the paper is to derive the regularity condition for asymmetry in EGARCH to provide the correct interpretation. It is shown that, in practice, EGARCH always displays asymmetry, though not leverage.

The paper is organized as follows. In Section 2, the GARCH and EGARCH models are derived from different underlying stochastic processes that lead to their derivation, the first from a random coefficient autoregressive process, and the second from a random coefficient complex nonlinear moving average process. Asymmetry and leverage are discussed in Section 3. The correct regularity condition for asymmetry in EGARCH is derived, and the correct interpretation is given, in Section 4. Some concluding comments are presented in Section 5.

2. Stochastic processes underlying GARCH and EGARCH

2.1. Random coefficient autoregressive process-GARCH

Consider the conditional mean of financial returns, as follows:

$$y_t = E\,(y_t|I_{t-1}) + \varepsilon_t, \tag{1}$$

where the financial returns, $y_t = \Delta \log P_t$, represent the log-difference in financial commodity prices, P_t, I_{t-1} is the information set at time $t-1$, and ε_t is a conditionally heteroskedastic error term, or returns shock. In order to derive conditional volatility specifications, it is necessary to specify the stochastic processes underlying the returns shocks, ε_t.

Now consider the random coefficient AR(1) process underlying the return shocks, ε_t:

$$\varepsilon_t = \phi_t \varepsilon_{t-1} + \eta_t \tag{2}$$

where

$\phi_t \sim iid\,(0, \alpha)\,, \alpha \geq 0,$
$\eta_t \sim iid\,(0, \omega)\,, \omega \geq 0,$
$\eta_t = \varepsilon_t/\sqrt{h_t}$ is the standardized residual, with h_t defined below.

Tsay (1987) derived the ARCH(1) model of Engle (1982) from Eq. (2) as:

$$h_t \equiv E\left(\varepsilon_t^2|I_{t-1}\right) = \omega + \alpha\varepsilon_{t-1}^2 \tag{3}$$

where h_t represents conditional volatility, and I_{t-1} is the information set available at time $t-1$. A lagged dependent variable, h_{t-1}, is typically added to Eq. (3) to improve the sample fit:

$$h_t \equiv E\left(\varepsilon_t^2|I_{t-1}\right) = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}. \tag{4}$$

From the specification of Eq. (2), it is clear that both ω and α should be positive as they are the unconditional variances of two different stochastic processes. Moreover, as GARCH is symmetric, there is no asymmetry or leverage.

Given the non-normality of the returns shocks, the Quasi-Maximum Likelihood Estimators (QMLE) of the parameters have been shown to be consistent and asymptotically normal in several papers. For example, Ling and McAleer (2003) showed that the QMLE for a generalized ARCH(p, q) (or GARCH(p, q)) is consistent if the second moment is finite. A sufficient condition for the QMLE of GARCH(1, 1) in Eq. (4) to be consistent and asymptotically normal is $\alpha + \beta < 1$.

2.2. Random coefficient complex nonlinear moving average process-EGARCH

A conditional volatility model that can accommodate asymmetry is the EGARCH model of Nelson (1990, 1991). McAleer and Hafner (2014) showed that EGARCH, specifically EARCH(1) = EGARCH(1, 0), could be derived from a random coefficient complex nonlinear moving average (RCCNMA) process, as follows:

$$\varepsilon_t = \phi_t\sqrt{|\eta_{t-1}|} + \psi_t\sqrt{\eta_{t-1}} + \eta_t \tag{5}$$

where

$\phi_t \sim iid\,(0, \alpha),$
$\psi_t \sim iid\,(0, \gamma),$
$\eta_t \sim iid\,(0, \omega),$
$\sqrt{\eta_{t-1}}$ is a complex-valued function of η_{t-1},
and $\eta_t = \varepsilon_t/\sqrt{h_t}$ is the standardized residual.

McAleer and Hafner (2014) show that the conditional variance of the squared returns shocks in Eq. (5) is:

$$h_t = E\left(\varepsilon_t^2|I_{t-1}\right) = \omega + \alpha|\eta_{t-1}| + \gamma\,\eta_{t-1}, \tag{6}$$

where it is clear from the RCCNMA process in Eq. (5) that all three parameters should be positive as they are the variances of three different stochastic processes. The constant ω is not equivalent to the unconditional variance of η_t. In Eq. (6), it is assumed that $h_t > 0$ as η_{t-1} could be negative, which is equivalent to assuming $\alpha > \gamma$. This is not a problem when the logarithmic approximation is used in Eq. (7).

Although the transformation of h_t in Eq. (6) is not logarithmic, the approximation given by:

$$\log h_t = \log\ (1 + (h_t - 1)) \approx h_t - 1$$

can be used to replace h_t in Eq. (6) with $1 + \log h_t$ to lead to EARCH(1) = EGARCH(1, 0):

$$\log h_t = E\left(\varepsilon_t^2|I_{t-1}\right) = (\omega - 1) + \alpha|\eta_{t-1}| + \gamma\,\eta_{t-1}, \tag{7}$$

The use of an infinite lag for the RCCNMA process in Eq. (5) would yield the standard EGARCH model with lagged conditional volatility, namely EGARCH(1, 1):

$$\log\ h_t = E\left(\varepsilon_t^2|I_{t-1}\right) = (\omega - 1) + \alpha|\eta_{t-1}| + \gamma\,\eta_{t-1} + \beta\ \log\ h_{t-1}. \tag{8}$$

As EGARCH can be derived from a random coefficient complex nonlinear moving average (RCCNMA) process, it follows from the specification in Eq. (5) that there is no invertibility condition to transform the returns shocks, ε_t, to the standardized residuals, η_t.