Economics Letters 161 (2017) 86–89

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

A social interaction model with ordered choices[☆]

Xiaodong Liu [*], Jiannan Zhou

Department of Economics, University of Colorado, Boulder, CO 80309, USA

HIGHLIGHTS

- We propose a social interaction model with ordered choices.
- The model has a micro-foundation based on an incomplete information game.
- Sufficient existence condition for a unique equilibrium of game is characterized.
- Identification and estimation by NFXP and NPL algorithms are discussed.
- Finite sample performance of the estimation methods is investigated.

ARTICLE INFO

Article history:
Received 14 September 2017
Received in revised form 20 September 2017
Accepted 22 September 2017
Available online 2 October 2017

JEL classification:
C31
C35

Keywords:
Ordered probit and logit models
Rational expectations
Social networks

ABSTRACT

We introduce a social interaction model with ordered choices. We provide a micro-foundation for the econometric model based on an incomplete information network game, and characterize the sufficient condition for the existence of a unique equilibrium of the game. We discuss the identification of the model, and propose to estimate the model by the NFXP and NPL algorithms. We conduct Monte Carlo simulations to investigate the finite sample performance of these two estimation methods.

1. Introduction

Estimation and inference methods have been proposed for social interaction models, with binary choices (e.g., Brock and Durlauf, 2001; Lee et al., 2014; Lin and Xu, 2016), unordered multinomial choices (e.g., Brock and Durlauf, 2002, 2006; Xu, forthcoming), and continuous choice variables (e.g., Bramoullé et al., 2009; Lee et al., 2010; Liu and Lee, 2010). However, little work has been done on the analysis of network models with ordered multinomial choices. Network data are usually from surveys, and thus the response variable of an empirical study is often ordinal in nature. For example, in the widely used National Longitudinal Study of Adolescent Health (Add Health) data,[1] academic performance in a certain subject ("A", "B", "C", "D or lower"), study effort ("I try very hard to do my best", "I try hard enough, but not as hard as I could", "I don't try very hard", "I never try at all"), and smoking and drinking frequency ("never", "once or twice", "once a month or less", "2 or 3 days a month", "once or twice a week", "3 to 5 days a week", "nearly everyday") are all coded as ordinal variables.

In this paper, we introduce a social interaction model with ordered choices. We provide a micro-foundation for the econometric model based on an incomplete information network game. We characterize the sufficient condition for the existence of a unique rational expectation equilibrium of the game, which in turn guarantees the coherency and completeness of the econometric model. We discuss the identification and estimation of the econometric model. As the econometric model involves the equilibrium rational expectation that needs to be solved from a fixed point mapping, it can be estimated by the nested fixed point (NFXP)

☆ We would like to thank the editor Badi H. Baltagi and an anonymous referee for helpful comments. All remaining errors are our own.

* Correspondence to: Department of Economics, University of Colorado Boulder, UCB 256, Boulder, CO 80309, USA.

E-mail address: xiaodong.liu@colorado.edu (X. Liu).

[1] For more information about the Add Health data, see http://www.cpc.unc.edu/projects/addhealth.

https://doi.org/10.1016/j.econlet.2017.09.027

algorithm (Rust, 1987) or the nested pseudo likelihood (NPL) algorithm (Aguirregabiria and Mira, 2007). We investigate the finite sample performance of these two estimation methods in Monte Carlo simulations. We find the NFXP estimator has a smaller standard deviation, while the NPL estimator is computationally more efficient.

The rest of the paper is organized as follows. Section 2 presents the econometric model and provides the sufficient condition for the existence of a unique equilibrium of the underlying network game. Section 3 discusses the identification of the model. Section 4 explains how to estimate the model parameters by the NFXP and NPL algorithms and how to interpret parameter estimates in terms of marginal effects. Section 5 provides simulation results on the finite sample performance of the proposed NFXP and NPL estimators. Section 6 concludes. The proofs are collected in the online appendix.

2. Model

A set of individuals $\mathcal{N} = \{1, \ldots, n\}$ interacts within a network. Let $\mathbf{W} = [w_{ij}]$ be an $n \times n$ predetermined adjacency matrix, where the (i, j)th element w_{ij} ($w_{ij} \geq 0$) captures the proximity of individuals i and j in the network. As a normalization, $w_{ii} = 0$ for all i. We assume the network topology captured by $\mathbf{W}$ is common knowledge among all individuals in the network.

Suppose all individuals in the network face m ordered alternatives. Individual i chooses alternative k, i.e. $y_i = k$, if and only if

$$\alpha_{k-1} < y_i^* \leq \alpha_k$$

where

$$y_i^* = \lambda \sum_{j=1}^{n} w_{ij} y_j^{\mathrm{E(i)}} + \mathbf{x}_i'\boldsymbol{\beta} + \epsilon_i \tag{1}$$

and α_k's are threshold parameters such that $-\infty = \alpha_0 < \alpha_1 < \cdots < \alpha_{m-1} < \alpha_m = \infty$. In Eq. (1), $y_j^{\mathrm{E(i)}}$ denotes individual i's *subjective* expected value of y_j. $\sum_{j=1}^{n} w_{ij} y_j^{\mathrm{E(i)}}$ is the weighted sum of individual i's subjective expectations on her peers' choices, and the coefficient λ represents the *peer effect*. $\mathbf{x}_i$ is a column vector of exogenous variables that captures the characteristics of individual i and her peers. For instance, let $\bar{\mathbf{X}}$ be an $n \times q$ matrix of observations on exogenous individual characteristics. Then, a possible specification of $\mathbf{X} = [\mathbf{x}_1, \ldots, \mathbf{x}_n]'$ is given by $\mathbf{X} = [\bar{\mathbf{X}}, \mathbf{W}\bar{\mathbf{X}}]$, with the coefficients of $\mathbf{W}\bar{\mathbf{X}}$ representing *exogenous contextual effects* (Manski, 1993). ϵ_i is a random shock that is independent of $\mathbf{W}$ and $\mathbf{X}$ and is independent and identically distributed (i.i.d.) with the distribution function $F(\cdot)$. Only individual i observes ϵ_i while everyone in the network observes $\mathbf{X}$. Then, given $\mathbf{W}$ and $\mathbf{X}$, the probability that individual i chooses alternative k is[2]

$$\Pr(y_i = k|\mathbf{W}, \mathbf{X}) = F(\alpha_k - \lambda \mathbf{w}_i \mathbf{y}^{\mathrm{E(i)}} - \mathbf{x}_i'\boldsymbol{\beta}) - F(\alpha_{k-1} - \lambda \mathbf{w}_i \mathbf{y}^{\mathrm{E(i)}} - \mathbf{x}_i'\boldsymbol{\beta}),$$

for $k = 1, \ldots, m$, where $\mathbf{w}_i$ denotes the ith row of $\mathbf{W}$ and $\mathbf{y}^{\mathrm{E(i)}} = [y_1^{\mathrm{E(i)}}, \ldots, y_n^{\mathrm{E(i)}}]'$. Let y_i^{E} denote the mathematical expectation of y_i. Then,

$$y_i^{\mathrm{E}} \equiv \mathrm{E}(y_i|\mathbf{W}, \mathbf{X}) = \sum_{k=1}^{m} k \Pr(y_i = k|\mathbf{W}, \mathbf{X}) = m - \sum_{k=1}^{m-1} F(\alpha_k - \lambda \mathbf{w}_i \mathbf{y}^{\mathrm{E(i)}} - \mathbf{x}_i'\boldsymbol{\beta}).$$

In the rational expectation equilibrium (see, e.g., Brock and Durlauf, 2001; Lee et al., 2014), the subjective expectation coincides with the mathematical expectation such that $\mathbf{y}^{\mathrm{E(i)}} = \mathbf{y}^{\mathrm{E}}$, where $\mathbf{y}^{\mathrm{E}} = [y_1^{\mathrm{E}}, \ldots, y_n^{\mathrm{E}}]'$, for all i. Thus, in the equilibrium, the rational expectation is characterized by the following system of equations,

$$\mathbf{y}^{\mathrm{E}} = \vec{h}(\mathbf{y}^{\mathrm{E}}; \boldsymbol{\delta}), \tag{2}$$

where $\vec{h}(\mathbf{y}^{\mathrm{E}}; \boldsymbol{\delta}) = [h_1(\mathbf{y}^{\mathrm{E}}; \boldsymbol{\delta}), \ldots, h_n(\mathbf{y}^{\mathrm{E}}; \boldsymbol{\delta})]'$ with

$$h_i(\mathbf{y}^{\mathrm{E}}; \boldsymbol{\delta}) = m - \sum_{k=1}^{m-1} F(\alpha_k - \lambda \mathbf{w}_i \mathbf{y}^{\mathrm{E}} - \mathbf{x}_i'\boldsymbol{\beta}),$$

and $\boldsymbol{\delta} = (\alpha_1, \ldots, \alpha_{m-1}, \lambda, \boldsymbol{\beta}')'$. A sufficient condition for the existence of a unique solution to the system of Eqs. (2) is given as follows. For an $n \times m$ matrix $\mathbf{A} = [a_{ij}]$, let the row sum and column sum matrix norms of $\mathbf{A}$ be denoted by $\|\mathbf{A}\|_\infty = \max_{i=1,\ldots,n} \sum_{j=1}^{m} |a_{ij}|$ and $\|\mathbf{A}\|_1 = \max_{j=1,\ldots,m} \sum_{i=1}^{n} |a_{ij}|$ respectively.

Assumption 1. (i) $\epsilon_1, \ldots, \epsilon_n$ are random shocks that are independent of $\mathbf{W}$ and $\mathbf{X}$ and are i.i.d. with the continuous distribution function $F(\cdot)$ and density function $f(\cdot)$. (ii) $|\lambda| < [(m-1)\sup_u f(u) \min\{\|\mathbf{W}\|_\infty, \|\mathbf{W}\|_1\}]^{-1}$.

If $\mathbf{W}$ is row-normalized with $\sum_{j=1}^{m} w_{ij} = 1$ for all i, then $\|\mathbf{W}\|_\infty = 1$. If ϵ_i follows the standard normal distribution, then $\sup_u f(u) = 1/\sqrt{2\pi}$. If ϵ_i follows the logistic distribution, then $\sup_u f(u) = 1/4$. Hence, with a row-normalized adjacency matrix, Assumption 1 holds for an ordered *probit* social interaction model if $|\lambda| < \sqrt{2\pi}/(m-1)$, and holds for an ordered *logit* social interaction model if $|\lambda| < 4/(m-1)$. It is worth noting that, when $m = 2$, Assumption 1 coincides with the sufficient condition for the existence of a unique rational expectation equilibrium of the social interaction model with binary outcomes in Lee et al. (2014).

Proposition 1. *Under Assumption 1, the social interaction model with ordered outcomes has a unique rational expectation equilibrium.*

When Assumption 1 holds, the contraction mapping property of $\vec{h}(\cdot)$ not only guarantees the coherency and completeness of the model (Tamer, 2003), but also suggests the system of Eqs. (2) can be solved by recursive iterations.

3. Identification

For identification, we follow Lee et al. (2014), Xu (forthcoming), Lin and Xu (2016) and others by assuming $F(\cdot)$ is a strictly increasing distribution function with unity variance that is known to the econometrician.[3] Furthermore, as in a standard ordered choice model (McKelvey and Zavoina, 1975), we impose an identification constraint that $\mathbf{X}$ does not have a constant column (i.e., the intercept is set to be zero).[4]

Given the network topology $\mathbf{W}$, two sets of parameters, $\boldsymbol{\delta} = (\alpha_1, \ldots, \alpha_{m-1}, \lambda, \boldsymbol{\beta}')'$ and $\widetilde{\boldsymbol{\delta}} = (\widetilde{\alpha}_1, \ldots, \widetilde{\alpha}_{m-1}, \widetilde{\lambda}, \widetilde{\boldsymbol{\beta}}')'$, are observationally equivalent if

$$\Pr(y_i \leq k|\mathbf{W}, \mathbf{X}) = F(\alpha_k - \lambda \mathbf{w}_i \mathbf{y}^{\mathrm{E}} - \mathbf{x}_i'\boldsymbol{\beta}) = F(\widetilde{\alpha}_k - \widetilde{\lambda} \mathbf{w}_i \widetilde{\mathbf{y}}^{\mathrm{E}} - \mathbf{x}_i'\widetilde{\boldsymbol{\beta}}),$$

2 Note that $F(\alpha_0 - \lambda \sum_{j=1}^{n} w_{ij} y_j^{\mathrm{E(i)}} - \mathbf{x}_i'\boldsymbol{\beta}) = 0$ and $F(\alpha_m - \lambda \sum_{j=1}^{n} w_{ij} y_j^{\mathrm{E(i)}} - \mathbf{x}_i'\boldsymbol{\beta}) = 1$.

3 In contrast, Brock and Durlauf (2007) consider the identification of binary choice group interaction models when the distribution of ϵ_i is unknown. In this paper, we discuss identification assuming $F(\cdot)$ is known as the proposed estimation procedure is parametric in nature.

4 Instead of dropping the intercept, one could impose an identification constraint that one of the threshold parameters α_k is a known constant (e.g., $\alpha_1 = 0$).