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A class of proximity-sensitive measures of relative deprivation

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HIGHLIGHTS

- A new class of generalized measures of relative deprivation is proposed.
- The class is characterized by a proximity-sensitive parameter p.
- Proximity is measured by the closeness of the incomes of higher-income individuals to the income of the reference individual.
- The class is capable of accommodating different weights accorded to changes in the incomes of higher-income individuals.

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ABSTRACT

We introduce a new class of generalized measures of relative deprivation. The class takes the form of a power mean of order p. A characteristic of the class is that depending on the value of the proximity-sensitive parameter p, the class is capable of accommodating both a decreasing weight (the case of $p > 1$), and an increasing weight (the case of $p \in (0,1)$) accorded to given changes in the incomes of the individuals who are wealthier than the reference individual, depending on their proximity in the income distribution to the reference individual.

1. Introduction

It is widely recognized that individuals feel stressed when their income (wealth) is lower than the income (wealth) of others with whom they naturally compare themselves (these "others" constitute the individuals' comparison group). The "relative deprivation" sensed by an individual can be measured in a variety of ways. The (income related) index that has become center stage is the aggregate of the excesses of the incomes of the other individuals in an individual's comparison group divided by the number of individuals in the individual's comparison group (essentially an operationalization of Runciman's 1966 relative deprivation concept by Yitzhaki, 1979; Hey and Lambert, 1980; Chakravarty, 1999; Ebert and Moyes, 2000; Bossert and D'Ambrosio, 2006; Stark and Hyll, 2011). An assumption made in both theoretical and empirical writings that have incorporated relative deprivation is that comparisons with others who are positioned to the right of the individual in the income distribution count equally: the income excesses of those who are close by and the income excesses of those who are farther away are accorded equal importance. However recent evidence (Obloj and Zenger, 2015; Quintana-Domeque and Wohlfart, 2016) indicates that people attach different importance to changes in incomes of individuals who are farther away in the income distribution than to changes in incomes of adjacent individuals.

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In this paper we question the equal weights convention. We propose a general and flexible weighting protocol, based on the notion that the same importance need not be attached to changes in income of individuals who are placed at different distances from the individual whose relative deprivation is measured. Operationalizing the income shortfall approach via a set of axioms enables us to obtain a class of measures that has the form of a power mean of the excesses of the incomes of others, parameterized by a positive number p.

Several other generalizations of the index of relative deprivation have already been proposed: Chakravarty and Chakraborty (1984), Paul (1991), Wang and Tsui (2000), Bossert and D'Ambrosio (2007, 2014), and Esposito (2010). The main difference between five of these six contributions and the generalization presented in this paper is that the indices proposed by Chakravarty and Chakraborty (1984), Paul (1991), and Wang and Tsui (2000) are not derived from axioms; the perspective pursued by Esposito (2010) is not based on the income shortfall; and the index proposed by Bossert and D'Ambrosio (2007) adheres to the equal weights convention. Only the generalization offered by Bossert and D'Ambrosio (2014) derives axiomatically a class of proximity-sensitive measures of relative deprivation based on income shortfalls. Our approach follows in the steps of Bossert and D'Ambrosio (2014), yet it takes the analysis a step further. Whereas the Bossert and D'Ambrosio's (2014) index allows for only one type of proximity-sensitivity, our proposed RD_p class of measures is proximity-sensitive in a more general sense: right-hand side changes in income weigh differentially, depending on how distant they are in the income distribution, and this variation is exhibited by the value of the proximity-sensitive parameter p: for $p\in(0,1)$, the greater the distance, the smaller the impact of a given change in income on the relative deprivation sensed by the individual; for $p>1$, the opposite effect applies.

As already noted, there can very well be situations in which people might be more disturbed by a given increase in income of an already relatively rich individual in their comparison group than by an equal increase in income of a not so rich individual in their comparison group. Thus, we derive a class of measures which, depending on the parameter p, can be applied to both types of sensitivity to the proximity of the incomes of others. Needless to say, the derived class of measures allows more nuanced analyses of settings in which relative deprivation considerations play a role. And, after all, if people need to be compensated for experiencing increased relative deprivation, the manner of calculating the index also matters greatly in the context of welfare-related policy formation.

In Section 2 we introduce a preference relation in the set of possible comparison groups, and we equip this relation with properties (axioms) that we consider natural for an ordering. We show that the only measure that fulfills the listed axioms is the index RD_p. In Section 3 we deal in some detail with the subset of the axioms that are related to the proximity-sensitivity property of RD_p. Section 4 concludes.

2. Axiomatization of order $p>0$ of the relative deprivation sensed by an individual

We consider a population of $n+1$ individuals, where n is a positive integer. The income distribution of this population is $(z,\mathbf{x})\in\mathbf{R}_+^{n+1}$, where z is the (non-negative) income of individual ω, and $\mathbf{x}=(x_1,\ldots,x_n)$ is the vector of (non-negative) incomes of the comparison group of ω. We denote $I_\mathbf{x}=\{i:x_i>z\}$, namely $I_\mathbf{x}$ is the subset of the comparison group $\mathbf{x}$ that consists of individuals whose incomes are higher than the income of ω. And we denote by Ω^{n+1} the set of vectors of (non-negative) incomes of individual ω and of the members of his comparison group: $(z,\mathbf{x})\in\Omega^{n+1}$.

We introduce a binary relation $\succeq$ on the set Ω^{n+1}. This relation will reflect an individual's preference for the level of relative deprivation arising from a comparison of his income z with the incomes of members of two different comparison groups: an individual will prefer a comparison group that makes him less relatively deprived. We denote by $\sim$ the symmetric part of $\succeq$, and by $\succ$ the asymmetric part of $\succeq$.

We begin with a set of axioms that are needed to ensure that comparisons with the incomes of other individuals are represented by non-negative income differences.

Focus axiom (Axiom F). Let $(z,\mathbf{x}),(z,\mathbf{y})\in\Omega^{n+1}$ be such that $I_\mathbf{x}=I_\mathbf{y}$ and $x_i=y_i$ for every $i\in I_\mathbf{x}$. Then $(z,\mathbf{x})\sim(z,\mathbf{y})$.

The Focus axiom requires the individual to be indifferent to the incomes of those who are poorer than him. The axiom reflects the fact that individual ω experiences relative deprivation only when he compares his income with incomes that are higher than his.

Translation Invariance axiom (Axiom TI). If $(z,\mathbf{x})\in\Omega^{n+1}$ and $\delta\in[-\min\{z,x_1,\ldots,x_n\},\infty)$, then

$$(z,x_1,\ldots,x_n)\sim(z+\delta,x_1+\delta,\ldots,x_n+\delta).$$

Translation Invariance requires the index of relative deprivation to be indifferent to a positive transformation, applied to all incomes, provided that all incomes stay non-negative. Therefore, the axiom imposes a sensitivity of the relative deprivation measure not to the absolute income of an individual, but to the income differences between the incomes of others and his own income.

Monotonicity axiom (Axiom M). Let $\mathbf{x}=(x_1,\ldots,x_i,\ldots,x_n)$ and $\mathbf{y}=(x_1,\ldots,x_i+\eta,\ldots,x_n)$ for some $i\in\{1,\ldots,n\}$ and $\eta>0$. Then, if $x_i+\eta>z$, we have that $(z,\mathbf{x})\succ(z,\mathbf{y})$.

The Monotonicity axiom requires an individual to be strictly more relatively deprived if a wealthier individual (meaning an individual whose income is higher) in his comparison group is made richer, and equally relatively deprived if a poorer individual is made richer yet remains (weakly) poorer. In addition by Axiom M, the larger the increase of the income of the wealthier individual, the larger the added relative deprivation experienced by individual ω.

Continuous Ordering axiom (Axiom CO). The relation $\succeq$ is a continuous linear ordering on Ω^{n+1} that can be represented by a continuous function (in the Euclidean metric on $\mathbf{R}^{n+1}$) $F:\Omega^{n+1}\mapsto[0,\infty)$ well-defined for all vectors $(z,\mathbf{x})\in\Omega^{n+1}$, that is,

$$(z,\mathbf{x})\succeq(z,\mathbf{y})\Leftrightarrow F(z,\mathbf{x})\le F(z,\mathbf{y}).$$

Axiom CO requires the binary relation to be a continuous linear ordering that is represented by a continuous function that, in turn, is well-defined for all possible income distributions. To ensure focus on essentials, in the remainder of this paper we draw on this representation, thereby bypassing the need to recall Axiom CO explicitly.

Reflexivity axiom (Axiom R). If all the components of the vector $\mathbf{x}$ are equal, that is, if $\mathbf{x}=(x,\ldots,x)$, then $F(z,\mathbf{x})=\max\{x-z,0\}$.

The Reflexivity Axiom requires that if individual ω compares his income with the incomes of the members of an "egalitarian" comparison group, then his relative deprivation with respect to this group is equal to the group's common income minus his own income, with a floor of zero.

Anonymity axiom (Axiom A). If $\mathbf{y}$ is a vector of incomes obtained from vector $\mathbf{x}$ by permutation of its components, then $(z,\mathbf{x})\sim(z,\mathbf{y})$.

The Anonymity axiom requires the binary relation to be indifferent to a permutation of the components of the reference vector. Thus, the axiom postulates an irrelevance of individual identities for the value of the index of relative deprivation.

Population Substitution Principle axiom (Axiom PSP). If $\mathbf{x}=(x_1,\ldots,x_n)$ and $(z,\mathbf{x})\in\Omega^{n+1}$, then $(z,x_1,\ldots,x_n)\sim(z,F(z,x_1\ldots,x_k)+z,\ldots,F(z,x_1\ldots,x_k)+z,x_{k+1},\ldots,x_n)$ for every $k\le n$.