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Significance test in nonstationary logit panel model with serially correlated dependent variable

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HIGHLIGHTS

- Nonstationary panel logit with serially correlated dependent variable is analyzed.
- The limit distribution of LM statistic is shown proportional to Chi-square.
- Significance test ignoring the serial correlation can result in spurious logit link.

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ABSTRACT

We derive the asymptotic distribution of the overall significance/LM test in logit panel models with nonstationary covariates when the binary dependent variable is serially correlated. The asymptotic distribution of LM statistic is shown proportional to Chi-square distribution. Spurious logit link could arise if one fails to take into account the serial correlation.

1. Introduction

Panel data modeling for binary dependent variable is a widely-used tool for empirical economic research. The 0–1 event indicator, say financial crisis or middle income trap in cross-country analysis, is often modeled with some relevant panel macroeconomic fundamentals such as GDP or investment. As the time-series components may exhibit nonstationarity, Park and Phillips (2000) develop new limit theories for the maximum likelihood estimator (MLE) in nonstationary time series binary choice models. They find that the Wald statistics still obey the χ^2 distribution asymptotically. Hence standard statistical inference can proceed in the usual manner. Two extensions of Park and Phillips (2000) are, Hu and Phillips (2004) which studies nonstationary time series ordered response models, and Jin (2009) that investigates discrete choice models in nonstationary panels.

The key assumption of the above papers is that $y_t = F\left(x_t'\beta\right)$ is correctly specified with $\beta \neq 0$, while empirically we may be interested in $H_0 : \beta = 0$. Therefore, it is useful to consider the distribution of MLE under$\beta = 0$, so that the overall significance test in a nonstationary binary model can be conducted. Guerre and Moon (2002) consider this case in time series setting with i.i.d binary choice and prove that the t-statistics obey standard normal distribution asymptotically. While i.i.d is convenient, a discrete time series is often autocorrelated. Chu et al. (2016) study the overall significance test in nonstationary multinomial logit model with serially correlated qualitative response.

In this letter we derive the overall significance test in nonstationary logit panel models when binary dependent variable is serially correlated. Discrete autoregression (DAR) process, introduced by Jacobs and Lewis (1978), is used to characterize such panel binary dependent variable. This particular characterization allows us to quantify the effect of serial correlation on the overall significance test. Hence the test can be correctly implemented with an easy adjustment.

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Table 1
Empirical sizes (%) of LM test ($N = 100$).

	$T = 30$	$T = 40$	$T = 50$	$T = 80$	$T = 100$	$T = 200$
p = 0.1	7.09	6.90	6.93	7.47	7.65	8.50
p = 0.3	11.20	13.79	13.31	14.20	15.20	15.00
p = 0.5	21.75	22.62	22.99	24.91	25.80	26.35

* Nominal size is 5%.
* The number of replications is 5000.

Our results in contrast to Jin (2009), rest in that panel binary dependent variable under consideration is serially correlated and it is possible to find a spurious logit link in empirical applications if one fails to take into account such serial correlation. Riddel (2003) also documents such a spurious problem using time series data but offers no theoretical justifications.

2. Main results

We begin with DAR process of order one to specify the binary panel data $\{y_{it}\}$,

$$y_{it} = \omega_{it} y_{i,t-1} + (1 - \omega_{it}) u_{it} \tag{1}$$

$$x_{it} = x_{i,t-1} + v_{it}$$

$$x_{i0} = 0, y_{i0} = 0, \quad i = 1, 2, \ldots, N, t = 1, 2, \ldots, T,$$

where ω_{it} and u_{it} are independent Bernoulli random variables with $\Pr(\omega_{it} = 1) = p_i$, and $\Pr(u_{it} = 1) = q_i$ for $i = 1, 2, \ldots, N$. It is easy to verify that $corr\left(y_{it}, y_{i,t-1}\right) = p_i$ and $E(y_{it}) = q_i$.

Suppose that y_{it} and x_{it} are independently generated, and a logit link with covariate x_{it} is used to model y_{it}, i.e. $\Lambda\left(\alpha_i + x_{it}'\beta\right) = \frac{e^{\alpha_i + x_{it}'\beta}}{1+e^{\alpha_i + x_{it}'\beta}}$, $i = 1, 2, \ldots, N$, $t = 1, 2, \ldots, T$. Then the conventional χ^2significance test against $H_0 : \beta = 0$ will reject the null too often and the size distortion gets severer when the autocorrelation of y_{it} increases. Table 1 provides Monte Carlo evidence for this claim (assume that $p_i = p$ for all i, q_i is randomly drawn from [0.2, 0.8] and v_{it} is i.i.d N(0,1)).

One may wonder whether he can adjust test statistics based on heteroskedasticity autocorrelation (HAC) method. Vogelsang (2012) studies "averages of HACs" standard errors that are robust to serial correlation including the nonstationary case for linear panel models. Similar to Vogelsang (2012), we construct "averages of HACs" standard errors for nonstationary panel logit. However, simulation results (available on request) suggest that this method can only partially remove the size distortion and its performance is even worse than non-adjusted standard statistics when correlation is low. We thus consider a different approach that uses the parametric nature of the model above to adjust the test statistic.

From (1), we have $E\left(y_{it}|I_{i,t-1}\right) = p_i y_{i,t-1} + (1 - p_i) q_i$. Define $\eta_{it} = y_{it} - E\left(y_{it}|I_{i,t-1}\right)$ and rewrite DAR process (1) in regression form:

$$y_{it} = m_i + p_i y_{i,t-1} + \eta_{it}, \tag{2}$$

$$\text{where } m_i = (1 - p_i) q_i, \eta_{it} \sim D\left(0, \sigma_i^2\right) \text{ and}$$
$$\sigma_i^2 = \left(1 - p_i^2\right) q_i (1 - q_i). \tag{3}$$

As $0 < p_i < 1$ for all i, $y_{it} = (1 - p_i L)^{-1} m_i + (1 - p_i L)^{-1} \eta_{it} = q_i + (1 - p_i L)^{-1} \eta_{it}$.

We thus define a moving average process

$$e_{it} = y_{it} - q_i = (1 - p_i L)^{-1} \eta_{it}. \tag{4}$$

In what follows we rely heavily on the panel functional central limit theorem (FCLT) developed in Phillips and Moon (1999).

Assumption 1 (*FCLT*)**.** Let $x_{it} = x_{i,t-1} + v_{it}$, where the $k \times 1$ error vector v_{it} is generated by the random coefficient linear process, suitably restricted to guarantee $\frac{1}{\sqrt{T}} x_{i,[Tr]}$ weakly converges to a randomly scaled Brownian motion with long-run conditional covariance matrix $c_i c_i'$ for all i, i.e. $\frac{1}{\sqrt{T}} x_{i,[Tr]} \to c_i B_i(r)$, as $T \to \infty$, $\forall i$. $B_i(r)$ is a k-dimensional standard Brownian motion.

Remark. Our aim is to apply this celebrated FCLT. We shall not repeat those suitable restrictions on the random coefficients detailed in Phillips and Moon (1999).

Assumption 2.

$\Pr(\omega_{it} = 1) = p$; $\Pr(u_{it} = 1) = q_i$, $i = 1, \ldots, N$,
$\omega_{it} \perp u_{it}$.

Remark. One can assume that p varies across i and obeys some distribution independent of v_{js}, ω_{it} and u_{it}. We shall discuss this case later.

Assumption 3. $v_{js} \perp (\omega_{it}, u_{it})$ for any given i, j, t, s.

Consider the log-likelihood in a fixed effect logit model:

$$l_{NT}(\alpha, \beta) = \sum_{i=1}^{N} \sum_{t=1}^{T} y_{it} \Lambda\left(\alpha_i + x_{it}'\beta\right) + \sum_{i=1}^{N} \sum_{t=1}^{T} (1 - y_{it}) \left(1 - \Lambda\left(\alpha_i + x_{it}'\beta\right)\right). \tag{5}$$

Let $\widetilde{\theta} = \begin{pmatrix} \widetilde{\alpha} \\ \widetilde{\beta} \end{pmatrix}$ and $\widetilde{\alpha} = (\widetilde{\alpha}_1, \widetilde{\alpha}_2, \ldots, \widetilde{\alpha}_N)'$ be the MLE that maximizes (5) under the null hypothesis $H_0 : \beta = 0$. The LM test is:

$$LM = S_{NT}'\left(\widetilde{\theta}\right) \left(-J_{NT}\left(\widetilde{\theta}\right)\right)^{-1} S_{NT}\left(\widetilde{\theta}\right),$$

where $S_{NT}(\cdot)$ and $J_{NT}(\cdot)$are the score and hessian, $\Lambda(z) = \frac{e^z}{1+e^z}$, $\dot{\Lambda}(z) = \Lambda(z)(1 - \Lambda(z))$. Hence $S_{NT}\begin{pmatrix} \widetilde{\alpha} \\ 0 \end{pmatrix}$ and $J_{NT}\begin{pmatrix} \widetilde{\alpha} \\ 0 \end{pmatrix}$ are of interest. See Box I.

Lemma 1.

(a) $$\frac{1}{T} \sum_{t=1}^{T} x_{it} e_{it} \overset{T\to\infty}{\to} \frac{\sigma_i}{1 - p} c_i \int_0^1 B_i(r) \, dB_{e_i}(r),$$

where $B_{e_i}(r)$ is standard Brownian motion and independent with $B_j(r)$, $i, j = 1, 2, \ldots, N$.

(b) $$\frac{1}{T^{3/2}} \sum_{t=1}^{T} x_{it} \overset{T\to\infty}{\to} c_i \int_0^1 B_i(r) \, dr,$$

(c) $$\frac{1}{T^2} \sum_{t=1}^{T} x_{it} x_{it}' \overset{T\to\infty}{\to} c_i \left(\int_0^1 B_i(r) B_i'(r) \, dr\right) c_i'.$$

Lemma 1 is a direct application of FCLT, we omit its proof. There are two ways to consider the convergence, sequential limit and joint limit. We consider only the sequential limit ($T \to \infty$, then $N \to \infty$) in this paper.

Lemma 2 (*Sequential Limit*)**.**

(a) $$\frac{1}{NT^2} \sum_{i=1}^{N} \sum_{t=1}^{T} x_{it} x_{it}' \overset{T\to\infty}{\to} \frac{1}{N} \sum_{i=1}^{N} c_i \left(\int_0^1 B_i(r) B_i'(r) \, dr\right) c_i' \underset{a.s.}{\overset{N\to\infty}{\to}} \frac{1}{2} V,$$