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Random categorization and bounded rationality[☆]

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HIGHLIGHTS

- We introduce a new stochastic choice rule, the Random Categorization (RCG) rule.
- We characterize the RCG in a stochastic choice dataset using an acyclicity axiom.
- The RCG accommodates violations of IIA (independence of irrelevant alternatives).

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ABSTRACT

In this study we introduce a new stochastic choice rule that categorizes objects in order to simplify the choice procedure. At any given trial, the decision maker deliberately randomizes over mental categories and chooses the best item according to her utility function within the realized consideration set formed by the intersection of the mental category and the menu of alternatives. If no alternative is present both within the considered mental category and within the menu the decision maker picks the default option. We provide the necessary and sufficient conditions that characterize this model in a complete stochastic choice dataset in the form of an acyclicity restriction on a stochastic choice revealed preference and other regularity conditions. We recover the utility function uniquely up to a monotone transformation and the probability distribution over mental categories uniquely. This model is able to accommodate violations of IIA (independence of irrelevant alternatives), of stochastic transitivity, and of the Manzini–Mariotti menu independence notion (i-Independence).

1. Introduction

Categorization has been recognized as an important part of the decision making process. Decision makers (DMs) categorize in order to simplify complex decision situations (Manzini and Mariotti, 2012). At the same time new evidence suggests that decision makers deliberately randomize when choosing (Agranov and Ortoleva, 2014). Here we provide a model that connects categorization, bounded rationality, and randomness in choice. The DM has access to a fixed set of categories defined as bundles of alternatives and, at any given trial, she considers with fixed probability one of those categories. Then the DM chooses according to her preferences the best item at the intersection of the considered category that is available in the menu. The probability of choosing a particular item in a menu is the sum of the probabilities of all mental categories that have a non-empty intersection with the menu and that, even more importantly, are such that there are no better alternatives within it than the fixed item.

This probabilistic categorization rule allows for menu dependence, and thus is more general than the popular model of limited attention and random choice put forward by Manzini and Mariotti (2014) (hereinafter MM). It also allows for degenerate probabilities and, in fact, the proposed model nests the standard rational model with strict preferences with a categorization rule that entails considering the category of all alternatives with probability 1. Stochastic intransitivity is also accommodated, as well as the similarity and compromise effects which represent a violation of the IIA condition (Luce, 1959 independence of irrelevant alternatives).

The random categorization (RCG) rule is characterized by the acyclicity of a stochastic revealed preference relation that consists of declaring a to be stochastically revealed preferred (strictly) to b if and only if the probability of choosing b in a menu is changed (either negatively or positively) by introducing a into such menu

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(i.e., if a has a non-zero impact on the probability of choosing b in a menu). The second condition is a total monotonicity requirement that is equivalent to the Block–Marschak conditions and thus makes our model a subcase of the random-utility model (RUM). The random categorization rule is distinct from other efforts to generalize MM and, in particular, it is not nested nor does it nest the random feasibility rule proposed by Brady and Rehbeck (2015) and Zhang (2016). All proofs are collected in Appendix.

2. The environment and the dataset

Formally, consider a finite choice set X. There is an always-available option $\{o\}$ (i.e., not choosing or a default that is always possible to obtain). A stochastic choice dataset is a set of menus $\mathcal{M} \subseteq 2^X$ together with a probabilistic choice map for any given menu and an item inside the menu: $p : X \cup \{o\} \times \mathcal{M} \to [0, 1]$, with $p(a, A)$ denoting the probability of picking a from $A \cup \{o\}$, and with $p(o, A)$ denoting the probability of not choosing anything from $A \in \mathcal{M}$, and thus picking the outside option or default. We fix $p(o, \emptyset) = 1$. The probability of choice is such that it adds-up to 1: $\sum_{a\in A} p(a, A)+p(o, A) = 1$. In summary, a dataset is the sequence of menus and choice probability pairs: $\{A, p(a, A)\}_{A\in\mathcal{M},a\in X\cup\{o\}}$. Henceforth, we require that the stochastic choice dataset be complete, equivalently, $\mathcal{M} \equiv 2^X$ is the power set.

3. The model: Random categorization (RCG) Rule

Having been given a menu, a DM who follows the random categorization rule selects a mental category with a fixed probability and then chooses the item that maximizes her utility from those alternatives that belong to the considered category and to the given menu. In the event that no item in the considered category is in the menu, the DM picks the default alternative $\{o\}$.

Formally, a DM is endowed with a collection of categories over the choice set X. We take the categories as given but we do not observe them. The categories are a collection of subsets of X, formally $\mathcal{D} \subseteq 2^X$. The DM has a probability measure that is defined over the categories and that represents her propensity to consider a given category at any given trial. A probability of consideration is a mapping $m : \mathcal{D} \mapsto [0, 1]$ such that $\sum_{D\in\mathcal{D}} m(D) = 1$ and $m(D) \in [0, 1]$. Finally, the DM also is endowed with a fixed utility function $u : X \mapsto \mathbb{R}$ that represents her tastes; we assume that it is injective or, equivalently, we rule out the possibility of indifference.

When facing a menu, the DM draws a mental category $D \in \mathcal{D}$ with probability $m(D)$ and then forms a consideration set $\Gamma(D, A) = D \cap A$. Then she picks $a = argmax_{b\in\Gamma(D,A)} u(b)$, which is the item that maximizes her utility in the consideration set. Thus under the RCG rule the probability of choosing $a \in A$ is given by $p_{RC}(a, A) = \sum_{D\cap A\neq\emptyset:D\in\mathcal{D}} \mathbb{I}(u(a) > u(b)\forall b \in (D \cap A) \setminus \{a\})m(D)$, where $\mathbb{I}(u(a) > u(b)\forall b \in (D \cap A) \setminus \{a\})$ is equal to 1 if the condition is true and is equal to zero if the condition is false. Alternatively, we can write $p_{RC}(a, A) = \sum_{\{a\}\cap D\neq\emptyset;\mathbf{B}_A(a)\cap D=\emptyset:D\in\mathcal{D}} m(D)$ where $\mathbf{B}_A(a) = \{b \in A : u(b) > u(a)\}$ is the set of better than a elements in the menu A.

Definition 1 (*Random Categorization rule, RCG*). A stochastic choice dataset has a Random Categorization rule representation if there is a triple u, m and $\mathcal{D}$ that are the injective utility function, the probability of consideration map, and the mental categories respectively, such that the probability of choosing $a \in X$ in a menu $A \in \mathcal{M}$, is the cumulative probability of all categories that produce a consideration set where $a \in A$ is the best element available:

$$p_{RCG}(a, A) = \sum_{\{a\}\cap D\neq\emptyset;\mathbf{B}_A(a)\cap D=\emptyset:D\in\mathcal{D}} m(D).$$

Finally, the probability of choosing the default is $p_{RCG}(o, A) = \sum_{A\cap D=\emptyset:D\in\mathcal{D}} m(D)$.

In summary, the probability of choosing $a \in A$ under the RCG rule $p_{RCG}(a, A)$ is equivalent to the probability that the DM considers $a \in A$ but does not consider any alternative that is better than it. By definition $\sum_{a\in A} p_{RCG}(a, A) + p_{RCG}(o, A) = 1$.

There are two important special cases of the RCG rule, namely the standard rational model (without indifference) and the MM model of consideration sets with menu independence.

Example 1 (*Standard Rational, DM*). A standard deterministic rational DM has a probability of choice $p_{DR}(a, A) = \mathbb{I}(u(a) > u(b)\forall b \in A \setminus \{a\})$, for an injective utility function $u : X \mapsto \mathbb{R}$ and for the indicator function $\mathbb{I}(\cdot)$. Clearly, this is a RCG rule with categories $\mathcal{D} = \{X\}$ with probability $m(X) = 1$ and with the same utility function u.

However, the RCG allows for deterministic revealed preferences reversals, or for violations of the Generalized Axiom of Revealed Preferences (GARP).

Example 2 (*Failures of Deterministic Rationality/Generalized Axiom of Revealed Preferences, GARP*). Consider $X = \{a, b, c\}$, and $\mathcal{D} = \{\{a, b\}, \{b, c\}, \{a, c\}\}$ such that $u(a) > u(b) > u(c)$. We have $m(\{i, j\}) = \frac{1}{3}$, for $i, j \in X$. Let $C_{p_{RCG}}(A) = \{a \in A | p_{RCG}(a, A) > 0\}$, be the choice correspondence induced by the RCG rule. Now, observe that $C_{p_{RCG}}(\{a, b, c\}) = \{a, b\}$ because $p_{RCG}(a, \{a, b, c\}) = \frac{2}{3}$ and $p_{RCG}(b, \{a, b, c\}) = \frac{1}{3}$. This means that we observe a to be (deterministically) strictly revealed preferred to c, aPc, with P representing a strict revealed preference relation that means that $a \in C(\{a, b, c\})$ while $c \notin C(\{a, b, c\})$. However, when we consider the menu $\{a, c\}$, we observe that $C_{p_{RCG}}(\{a, c\}) = \{a, c\}$ this means that c is revealed indifferent to a, cIa, where I is the deterministic revealed indifference relation such that $c \in C(\{a, c\})$ and $a \in C(\{a, c\})$. In summary, given that GARP fails, then there is no utility function such that the choice correspondence is generated by maximizing it: $C_{p_{RCG}}(A) = argmax_{a\in A} u(a)$.

The MM model is also a special case of the RCG rule.

Example 3 (*MM stochastic consideration with menu independence*). The MM model of consideration sets consists of an attention parameter $\gamma : X \mapsto (0, 1)$ and a utility function $u : X \mapsto \mathbb{R}$ such that $p_{MM}(a, A) = \gamma(a)\prod_{b\in A:u(b)>u(a)}(1 - \gamma(b))$. In this case, the categories are comprised of all possible subsets of X including the empty set (which has positive probability), $\mathcal{D} = 2^X$ and the probability of consideration of the categories is $m(D) = \sum_{A\subseteq D}(-1)^{|D\setminus A|}(\prod_{a\in X\setminus A}(1-\gamma(a)))$, which the reader can verify generates the following: $p_{RCG}(a, A) = \sum_{\{a\}\cap D\neq\emptyset;\mathbf{B}_A(a)\cap D=\emptyset:D\in\mathcal{D}} m(D) = \gamma(a)\prod_{b\in A:u(b)>u(a)}(1 - \gamma(b))$. The fact that γ is non-degenerate implies that the support of m is the whole power set 2^X, or alternatively that the categories include all of the elements of the power set.

Of course, the RCG rule, allows for probabilistic datasets that cannot be accommodated by neither the deterministic rational model or the MM model.

Example 4 (*Categorization/Failures of i-Independence*). Consider the choice set $X = \{a, b, c, d\}$, such that $u(c) > u(a) > u(b) > u(d)$ with categories $\mathcal{D} = \{\{a, c, b\}, \{a\}, \{b, d\}\}$. We let the map $m : \mathcal{D} \mapsto (0, 1)$ be any non-degenerate probability over the categories, for all $D \in \mathcal{D}$. Then we give the DM the menus $A = \{a, c, b\}$, $B = \{a, d, b\}$, and thus we have: (i) $p_{RCG}(a, A \setminus \{b\})/p_{RCG}(a, A) = m(\{a\})/m(\{a\})$, (ii) $p_{RCG}(a, B \setminus \{b\}) = m(\{a, c, b\}) + m(\{a\})$, and (iii) $p_{RCG}(a, B) = m(\{a\})$. Finally, (i), (ii) and (iii) imply that $\frac{p_{RCG}(a,A\setminus\{b\})}{p_{RCG}(a,A)} < \frac{p_{RCG}(a,B\setminus\{b\})}{p_{RCG}(a,B)}$, when $m(D) > 0$ for all $D \in \mathcal{D}$. Note that i-Independence, a necessary condition for MM, that requires that $\frac{p(a,A\setminus\{b\})}{p(a,A)} = \frac{p(a,B\setminus\{b\})}{p(a,B)}$ for all $a, b \in A \cap B$ is violated.