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A taxonomy of rationalization by incomplete preferences

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HIGHLIGHTS

- We provide a taxonomy of 4 known models of rationalization with (possibly) incomplete preferences.
- We provide a novel characterization of rationalizability that gives a new interpretation of WARP.
- Our taxonomy provides a novel axiomatic rationale for a moderate attraction effect.

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ABSTRACT

This paper provides a tight axiomatic relation between the following models: the standard rational choice model, non-domination rationalizability, rationalization by an incomplete Hype-relation (Aizerman and Malishevski, 1981), and the transitive version of the partial dominant choice model (Gerasimou, 2016). Thus, providing a taxonomy of rationalization by incomplete preferences.

1. Introduction

1.1. Background

The strictest notion of rationalization is the utility maximization choice rule, which is characterized, in the case of finite universal set of alternatives, by the famous weak axiom of revealed preference (WARP). A weaker form of rationalization is *non-domination (ND) rationalizability* under which the decision maker (DM) chooses all non-dominated alternatives according to a partial order. The choice theoretic foundations of ND-rationalization are studied, among others, by Bandyopadhyay and Sengupta (1993) and Eliaz and Ok (2006) who characterize it with an intuitive relaxation of WARP, called the weak axiom of revealed non-inferiority (WARNI). Gerasimou (2016) recently offered another notion of rationalization with incomplete preferences that provides a simple explanation for the attraction effect and in which the DM chooses only non-dominated options that dominate some available alternatives.[1] Aizerman and Malishevski (1981) offered another concept of rationalization by a hyper-relation (henceforth, HR-rationalization): an alternative x is chosen from a menu S if and only if the DM prefers x over S.

1.2. Results

The contribution of this paper is threefold. First, we recompose WARP from WARNI, which has the straightforward interpretation of consistency in rejection, and a novel axiom, called the weak axiom of revealed superiority (WARS), which has the matching interpretation of consistency in selection. This result provides us a new way of thinking of WARP as it is independent of Sen's (1971) division, who decomposed WARP into consistency in contraction and consistency in expansion.

[1] The attraction effect is a well-documented phenomenon, see Huber et al. (1982).

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Second, we moderately relax WARNI to allow for some non-dominated outcomes to be rejected. This generalization results with rationalization by (possibly) incomplete Hyper-relation.

Finally, we show that our relaxation of WARNI together with WARS characterizes the transitive version of Gerasimou's (2016) partially dominant choice model (henceforth, TPD).[2] In other words, we show that while TPD agent satisfies a moderately weak version of WARNI, it satisfies WARNI itself if and only if he is a utility maximizer. This gives the exact axiomatic relation between the standard utility maximizing choice rule, ND-rationalization, HR-rationalization, and TPD, and provide a sharp taxonomy of rationalization by incomplete preferences.

2. Preliminaries and definitions

Let X be a finite set of alternatives[3] and let χ be the set of all non-empty subsets of X. The elements of χ are viewed as feasible sets that a DM may need to choose an alternative from. A *choice correspondence* on χ is defined as any correspondence $c : \chi \twoheadrightarrow X$, such that $\emptyset \neq c(S) \subseteq S$ contains all the elements that the DM is willing to choose from S. To ease notation, we define $R \subseteq X \times X$ by the rule xRy if $\exists S \in \chi : x \in c(S)$ and $y \in S$.

The DM's *preferences* are denoted by a reflexive and transitive binary relation $\succeq$ on X, as usual $\succ$ and $\sim$ denote the asymmetric and symmetric parts of $\succeq$, respectively. The incomparable part of such a relation is denoted by $\bowtie$; that is, $x \bowtie y$ if and only if $x \not\succeq y$ and $y \not\succeq x$, where $\not\succeq$ is defined by $x \not\succeq y$ if $\neg(x \succeq y)$.[4]

Definition 1. A choice correspondence c on χ is non-domination rationalizable (ND) if for all $S \in \chi, c(S) = \{x | y \not\succ x \text{ for all } y \in S\}$, for some preorder $\succeq$.

That is, a choice correspondence is ND if it expresses incomplete preferences in the sense all $\succeq$-non-dominated outcomes are chosen from each choice problem, where $\succeq$ is some reflexive and transitive relation. If $\succeq$ is also complete, then c is said to be *complete preference rationalizable*.

Definition 2. A choice correspondence c on χ is complete preference rationalizable if for all $S \in \chi, c(S) = \{x \mid x \succeq y \text{ for all } y \in S\}$, for some complete and transitive preference relation $\succeq$.

Next, we provide the formal definition of (possibly incomplete) HR-rationalization. Let $\trianglerighteq \subseteq X \times \chi$ be a hyper-relation where $x \trianglerighteq A$ is interpreted as alternative x is preferred to facing menu A.

Definition 3. A choice correspondence c on χ is hyper-relation (HR) rationalizable if for all $S \in \chi, x \in c(S) \subset S$ iff $x \triangleright S$ for some partial order $\trianglerighteq$.

Recall that with no restriction on $\trianglerighteq$, any choice correspondence is HR-rationalization (Nehring, 1997). Thus, when $\trianglerighteq$ is complete, it is also assumed to be monotonic (e.g., Aizerman and Malishevski, 1981; Nehring, 1997), namely, for all $x \in S$ such that $x \trianglerighteq S$, $x \trianglerighteq S \cup \{y\}$ iff $x \trianglerighteq \{y\}$. Here, we impose the immediate generalization of monotonicity to incomplete preferences.

Definition 4. An hyper relation $\triangleright$ is monotonic if for all $x \in S$ such that $x \triangleright S$, $x \triangleright S \cup \{y\}$ iff $y \not\trianglerighteq \{x\}$ and for all $x \in S$ such that $x \sim S$, $x \trianglerighteq S \cup \{y\}$ iff $x \trianglerighteq \{y\}$.

We close this section with the definition of the TPD model.

Definition 5. A choice correspondence c on χ is called TPD if for all $S \in \chi, x \in c(S) \subset S$ iff $y \not\succ x \; \forall y \in S$ and $x \succ y$ for some $y \in S$, for some asymmetric and transitive preference relation $\succ$.

Remark 1. Since $c(S) \neq \emptyset$, it follows from Definition 3(5) that if $x \not\triangleright S$ for all $x \in S$ ($y \not\succ x, \forall x, y \in S$), then $c(S) = S$.

That is, an agent whose behavior is captured by the HR choice correspondence chooses from menu S all the options that are preferred to that menu, and an agent whose behavior is captured by the TPD choice correspondence chooses only non-dominated options that are strictly preferred to some available alternative. If no alternative in the menu dominates it, then HR agent chooses from each of the feasible alternatives, and similarly for a TPD agent when no alternative dominates another.

3. Results

It is well-known that the weak axiom of revealed preference is necessary and sufficient for a choice correspondence to be complete preference rationalizable (e.g., Sen, 1971).

Weak Axiom of Revealed Preference (WARP). For any $x, y \in X$, if for some $S \in \chi, x \in c(S)$ and $y \in S/c(S)$, then $y\not R x$

The intuition behind WARP is as follows: if y is rejected when x is chosen, then one may conclude that x is strictly preferred over y, and thus y will never be chosen when x is available.

We note that WARP is equivalent to each of the following conditions:

WARP'. For any $S \in \chi$ and $y \in X$, if $y \in S/c(S)$, then $y\not R x$ for all $x \in c(S)$.
WARP". For any $S \in \chi$ and $x \in X$, if $x \in c(S)$, then $y\not R x$ for all $y \in S/c(S)$.

Eliaz and Ok (2006) showed that the following straightforward weakening of WARP' is necessary and sufficient for ND-rationalizability.

Weak Axiom of Revealed Non-Inferiority (WARNI). For any $S \in \chi$ and $y \in X$, if $y \in S/c(S)$, then $y\not R x$ for some $x \in c(S)$.

WARNI relaxes WARP' so that if a choice object y is rejected from a menu S, then it will never be chosen when x is available for some x that is chosen from S (rather than for all elements chosen from S). The intuition behind this relaxation is that, since the underlying relation can be incomplete, rejecting y from S need not imply that y is inferior to all chosen elements from S, but rather that y can be inferior only to some elements in S.

The following axiom weakens WARP" in the same manner that WARNI weakens WARP'.

Weak Axiom of Revealed Superiority (WARS). For any $S \in \chi$ and any $x \in X$, if $x \in c(S) \subset S$, then $y\not R x$ for some $y \in S/c(S)$.

Similar to the justification of WARNI, relaxing WARP to WARS can be justified as follows: when having incomplete preference, choosing x (non-trivially) form menu S is consistent also with x being superior only to some rejected alternatives from S. Our first result states that WARNI and WARS together are equivalent to WARP.

Theorem 1. *Let c be a choice correspondence on χ. The following are equivalent:*[5]

[2] See Qin (2017) on the advantages of focusing on the transitive version of partial dominance.

[3] All the results can be easily generalized to the case in which the set of alternatives is a compact metric space. Theorem 1 holds also for an arbitrary set of alternatives.

[4] More generally, for a binary relation B, we write $x\not B y$. to indicate $(x, y) \notin B$.

[5] All proofs are in Appendix.