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Factor substitution and long-run growth in the Lucas model with elastic labor supply

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HIGHLIGHTS

- I examine the link between factor substitution and endogenous long-run growth.
- I use the Lucas model with physical and human capital and elastic labor supply.
- Positive link if the baseline capital–effective labor ratio is above its steady state.
- The relationship is negative otherwise.

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ABSTRACT

We show that there exists a positive (resp., negative) relationship between the elasticity of factor substitution and long-run growth if the baseline ratio of physical capital to effective labor is above (resp., below) its steady-state value in the Lucas model with elastic labor supply.

1. Introduction

The relationship between factor substitutability and growth has received much attention in the literature. The seminal contributions of de La Grandville (1989) and Klump and de La Grandville (2000) uncover the positive relation between the elasticity of substitution and growth in the Solow model. Klump (2001) shows that this relationship is positive in the Ramsey–Cass–Koopmans model if the baseline capital per capita is below its steady-state value but ambiguous otherwise. Miyagiwa and Papageorgiou (2003) show that such positive relationship does not necessarily hold in the Diamond overlapping-generations model either. Irmen and Klump (2009) reconcile these findings by introducing possible asymmetries of savings out of factor income. Xue and Yip (2012) present a comprehensive characterization of the link between the elasticity of substitution and the steady-state capital and output per capita in the Solow, Ramsey–Cass–Koopmans and Diamond models.

This literature has considered one-sector models in which long-run growth is exogenously given. As a consequence, factor substitution can affect transitional but not long-run growth. Furthermore, they consider that the factors of production are physical capital and (raw) labor rather than effective labor —i.e., labor adjusted for human capital. Our purpose is to overcome these limitations.

This paper studies the relationship between long-run growth and factor substitution in the Lucas (1988) two-sector endogenous growth model with physical and human capital. The fact that human capital accumulation depends only on effective time devoted to education allows us to focus on the growth effect of factor substitutability in the goods production sector. The long-run growth rate in the *standard* Lucas (1988) model with inelastic labor supply does not depend on the parameters of the goods production sector; in particular, the elasticity of substitution. Thus, we incorporate the labor-leisure choice to the model following Ladrón-de Guevara et al. (1999). We show that the relationship between the elasticity of factor substitution and long-run growth is positive (resp., negative) if the baseline ratio of physical capital to effective labor is above (resp., below) its steady-state value. This result contrasts with that

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obtained by Gómez (2015) in the one-sector endogenous growth with physical and human capital – and inelastic labor supply – where the nexus between factor substitution and long-run growth is positive irrespective of the relative positions of the baseline and the steady-state ratios of physical to human capital.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes the substitutability–growth nexus. Section 4 concludes.

2. The Lucas model with leisure

The economy is populated by a large number of identical infinitely-lived agents which, for simplicity, is normalized to unity. At each moment of time, the agent is endowed with a unit of time which can be devoted to goods production, u, studying, z, or leisure, l, so that $u+z+l=1$.

2.1. Firms

Output Y is produced using physical capital K and effective labor uH, where H denotes human capital, by means of the CES technology

$$Y=F(K,uH)=A[\alpha K^{\psi}+(1-\alpha)(uH)^{\psi}]^{1/\psi},$$
$$A>0,\ \ 0<\alpha<1,\ \ \psi<1,$$

where $\sigma = 1/(1-\psi)$ is the elasticity of substitution. Denoting $y=Y/(uH)$ and $k=K/(uH)$, the production function in intensive form can be written as

$$y=f(k)=F(k,1)=A[\alpha k^{\psi}+(1-\alpha)]^{1/\psi}.$$

Profit maximization entails that

$$r=f'(k)=\alpha A^{\psi}[f(k)/k]^{1-\psi}, \tag{1}$$
$$w=f(k)-kf'(k)=(1-\alpha)A^{\psi}f(k)^{1-\psi}, \tag{2}$$

where r is the interest rate and w is the wage rate.

2.2. Agents

The representative agent maximizes the utility derived from consumption C and leisure l,

$$U=\int_0^{\infty}(\ln C+v\ln l)e^{-\rho t}dt,\quad v>0,\ \rho>0, \tag{3}$$

subject to the budget constraint

$$\dot K=rK+wuH-C-\delta_K K,\quad \delta_K>0, \tag{4}$$

and the constraint on human capital accumulation

$$\dot H=\xi zH-\delta_H H,\quad \xi>0,\ \delta_H>0, \tag{5}$$

where δ_K and δ_H are the rates of depreciation of physical and human capital, respectively. The current-value Hamiltonian of the agent's problem is

$$\mathcal{H}=\ln C+v\ln l+\lambda(rK+wuH-C-\delta_K K)+\mu(\xi zH-\delta_H H).$$

Substituting $z=1-l-u$, the first-order conditions for an interior solution are

$$\partial\mathcal{H}/\partial C=1/C-\lambda=0, \tag{6}$$
$$\partial\mathcal{H}/\partial l=v/l-\lambda wH=0, \tag{7}$$
$$\partial\mathcal{H}/\partial u=(\lambda w-\mu\xi)H=0, \tag{8}$$
$$\dot\lambda=(\rho+\delta_K)\lambda-\partial\mathcal{H}/\partial K=(\rho+\delta_K-r)\lambda, \tag{9}$$
$$\begin{aligned}\dot\mu&=\rho\mu-\partial\mathcal{H}/\partial H=(\rho+\delta_H)\mu-\lambda wu-\mu\xi(1-u-l)\\&=[\rho+\delta_H-\xi(1-l)]\mu,\end{aligned} \tag{10}$$

where we have used (8) to get the last equality in (10), together with the transversality condition

$$\lim_{t\to\infty}e^{-\rho t}\lambda K=\lim_{t\to\infty}e^{-\rho t}\mu H=0. \tag{11}$$

2.3. Equilibrium

Ladrón-de Guevara et al. (1999, Theorem 1) prove that there always exists an optimal solution to the above problem. The system that drives the dynamics of the economy in terms of r, $c=C/K$ and l is

$$\dot r=-\left[\left(\frac{r}{\alpha A^{\psi}}\right)^{\frac{1}{1-\psi}}-r\right][r-\delta_K-\xi(1-l)+\delta_H], \tag{12}$$
$$\dot c=c\left[r-\left(\frac{r}{\alpha A^{\psi}}\right)^{\frac{1}{1-\psi}}+c-\rho\right], \tag{13}$$
$$\dot l=l\,[\xi u(r,c,l)-\rho], \tag{14}$$

where using (6) and (7), taking into account (1) and (2), we have that

$$u(r,c,l)=\frac{l}{vc}\left[\frac{f(k)-kf'(k)}{k}\right]=\frac{l}{vc}\left[\left(\frac{r}{\alpha A^{\psi}}\right)^{\frac{1}{1-\psi}}-r\right]. \tag{15}$$

This system is obtained as follows. Log-differentiating (7) and (2), using (9) and (10), we get respectively

$$\dot w/w=\dot\mu/\mu-\dot\lambda/\lambda=r-\delta_K-\xi(1-l)+\delta_H, \tag{16}$$
$$\dot w/w=\frac{-k\dot r}{f(k)-kf'(k)}=-\left[\left(\frac{r}{\alpha A^{\psi}}\right)^{\frac{1}{1-\psi}}-r\right]^{-1}\dot r. \tag{17}$$

Eq. (12) is obtained from (16) and (17). Log-differentiating (6) and using (9) we get that

$$\dot C/C=r-\rho-\delta_K=-\dot\lambda/\lambda. \tag{18}$$

The budget constraint (4) can be expressed as

$$\dot K/K=f(k)/k-c-\delta_K=\left(\frac{r}{\alpha A^{\psi}}\right)^{\frac{1}{1-\psi}}-c-\delta_K. \tag{19}$$

Now, Eq. (13) is obtained using that $\dot c/c = \dot C/C - \dot K/K$. Log-differentiating (7) we get that $\dot l/l=-(\dot\lambda/\lambda+\dot w/w+\dot H/H)$ which, using (5), (16) and (18), yields (14).

2.4. Steady state

Let $\bar\gamma$ be the (common) steady-state growth rate of consumption, physical capital and human capital. In the steady state we have that

$$\bar\gamma=\left(\frac{\bar r}{\alpha A^{\psi}}\right)^{\frac{1}{1-\psi}}-\bar c-\delta_K, \tag{20}$$
$$\bar\gamma=\xi(1-\bar u-\bar l)-\delta_H, \tag{21}$$
$$\bar\gamma=\bar r-\delta_K-\rho, \tag{22}$$
$$\bar\gamma=\xi(1-\bar l)-\delta_H-\rho, \tag{23}$$
$$v\frac{\bar c}{\bar l}\bar u=\left(\frac{\bar r}{\alpha A^{\psi}}\right)^{\frac{1}{1-\psi}}-\bar r=\bar c-\rho. \tag{24}$$

Eqs. (20), (21) and (22) result from (19), (5) and (18), respectively. Eq. (23) results from (12) and (22), and Eq. (24) results from (15) together with (13).

Using (23) and (21) we get that $\bar u = \rho/\xi$. The transversality condition (11) is equivalent to

$$\begin{aligned}-\rho+\rho+\delta_K-\bar r+\bar\gamma&=-\xi(1-\bar l)+\xi(1-\bar u-\bar l)\\&=-\xi\bar u=-\rho<0,\end{aligned}$$