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The semiparametric asymmetric stochastic volatility model with time-varying parameters: The case of US inflation

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HIGHLIGHTS

- A semiparametric asymmetric stochastic volatility model with time-varying parameters is considered.
- An efficient Markov Chain Monte Carlo estimation algorithm is developed.
- The proposed model is applied to inflation modeling.
- The proposed model shows positive correlation between inflation and volatility.
- The proposed model forecasts better that competing models.

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ABSTRACT

We propose a semiparametric extension of the time-varying parameter regression model with asymmetric stochastic volatility. For parameter estimation we use Bayesian methods. We illustrate our methods with an application to US inflation.

1. Introduction

Time varying-parameter regression models with stochastic volatility (TVP-SV models) have been successfully applied to inflation modeling (Stock and Watson, 2007; Clark and Ravazzolo, 2015; Chan, 2017).

In this paper, we focus on the relationship between inflation and volatility that has been examined by many researchers. Friedman (1977) points out the potential positive association between inflation and volatility. There are also many empirical evidences, including Baillie et al. (1996), Grier and Perry (1998) and Fountas (2001). Chan (2017) developed a stochastic volatility in mean model with time-varying parameters and applied it to estimate inflation. Chan (2017) found positive relationship between inflation and volatility before early 1980s, and zero or even negative after early 1980s.

The contribution of this paper is threefold. First, we capture the correlation between inflation and volatility by modeling jointly the distribution of inflation and log-volatility within a TVP-SV model. Furthermore, the joint distribution of inflation and volatilities is modeled semiparametrically. The intuition behind this semiparametric extension is that macroeconomic shocks that have the greatest effect on the economy are often not symmetric, suggesting that innovations have a distribution that is skewed to the left or to the right.

Dimitrakopoulos (2017) extended semiparametrically the TVP-SV model by using mixtures of Dirichlet processes (Ferguson, 1973) for the observations' errors and the errors of the parameter-driven dynamics. Dimitrakopoulos (2017)'s mixture approach over both the mixture's means and variances of the observation

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distribution can capture this skewness. An alternative flexible approach to capturing skewness is to jointly model nonparametrically the bivariate distribution of the observations and the log-volatilities. This approach was proposed by Jensen and Maheu (2014) who used a bivariate Dirichlet process mixture model for the innovations of a SV model with leverage to examine the behavior of daily returns.

Following Jensen and Maheu (2014), we extend the model of Dimitrakopoulos (2017) by accounting for a semiparametric asymmetric stochastic volatility that captures in a flexible way the joint distribution of the empirical skewness of inflation. The resulting model specification is novel and constitutes our second contribution.

We use Bayesian methods and develop an efficient Markov chain Monte Carlo algorithm for estimating the parameters of the model. This is our third contribution.

2. Econometric set up

2.1. The TVP-SV model with correlated errors

Consider the following time-varying parameter regression model with asymmetric stochastic volatility

$$y_t = \mu + \mathbf{x}_t'\boldsymbol{\beta} + \mathbf{z}_t'\boldsymbol{\alpha}_t + \exp(h_t/2)\varepsilon_t, t = 1, \ldots, T, \quad (1)$$

$$\boldsymbol{\alpha}_{t+1} = \boldsymbol{\alpha}_t + \mathbf{u}_t, \mathbf{u}_t \sim N(\mathbf{0}, \boldsymbol{\Sigma}), t = 0, 1, \ldots, T-1, \quad (2)$$

$$h_{t+1} = \mu_h + \phi(h_t - \mu_h) + \eta_t, |\phi| < 1, \quad (3)$$

where the errors ε_t and η_t are independently and identically distributed following the bivariate normal distribution,

$$\begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho\sigma_h \\ \rho\sigma_h & \sigma_h^2 \end{pmatrix}\right]. \quad (4)$$

In Eq. (1), μ is the intercept, $\boldsymbol{\beta}$ is the constant coefficient vector of dimension $k \times 1$ and $\boldsymbol{\alpha}_t$ are the time-varying coefficients of dimension $p \times 1$. No constant is included in the design matrices $\mathbf{x}_t$ and $\mathbf{z}_t$.

The parameter-driven dynamics in Eq. (2) follow a random walk process which is initialized with $\boldsymbol{\alpha}_0 = \mathbf{0}$ and $\mathbf{u}_0 \sim N(\mathbf{0}, \boldsymbol{\Sigma}_0)$, for known initial covariance matrix $\boldsymbol{\Sigma}_0$.

In Eq. (3), the term h_t is the log-volatility at time t and ϕ is a persistence parameter that satisfies the stationarity restriction ($|\phi| < 1$). The AR(1) stochastic volatility process is initialized with $h_1 \sim N(\mu_h, \sigma_h^2/(1-\phi^2))$.

The model given by expressions (1)–(4) is the TVP-SV model with correlated errors[1] (TVP-SVC model). Furthermore, when the correlation parameter ρ equals zero, the TVP-SVC model reduces to the standard TVP-SV model.

We also assume the following priors

$$\boldsymbol{\beta} \sim N(\boldsymbol{\beta}_0, \mathbf{B}),\ \sigma_h^2 \sim \mathcal{IG}(v_a/2, v_\beta/2),\ \boldsymbol{\Sigma} \sim IW(\delta, \Delta^{-1}),$$
$$\mu_h \sim N(\bar{\mu_h}, \bar{\sigma_h}^2),\ \ \mu \sim N(\bar{\mu}, \bar{\sigma}^2), \rho \sim N(\rho_0, \sigma_\rho^2)I_{|\rho|<1},$$
$$\phi \sim N(\phi_0, \sigma_\phi^2)I_{|\phi|<1},$$

where IW and $\mathcal{IG}$ denote the Inverse-Wishart distribution and the inverse gamma distribution, respectively. $I_{|\rho|<1}$ is an indicator function that equals one for the stationary region and zero otherwise and $N(\rho_0, \sigma_\rho^2)I_{|\rho|<1}$ is a normal density truncated in the stationary region. Similar analysis holds for the prior of ϕ.

[1] In finance, the negative correlation between ε_t and η_t is called leverage effect: as asset prices decline, companies become mechanically more leveraged since the relative value of their debt rises relative to that of their equity. As a result, it is natural to expect that their stock becomes riskier, hence more volatile. It is difficult to imagine that a similar economic argument exists for inflation. For this reason, we avoid using the term "leverage" throughout the paper.

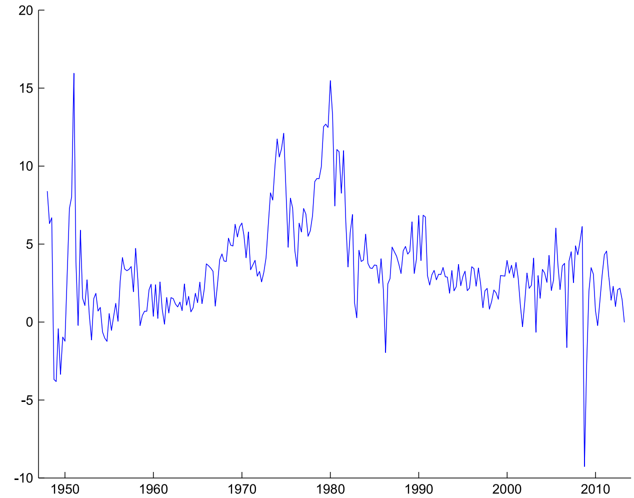

Fig. 1. The inflation path from 1948Q1 to 2013Q2.

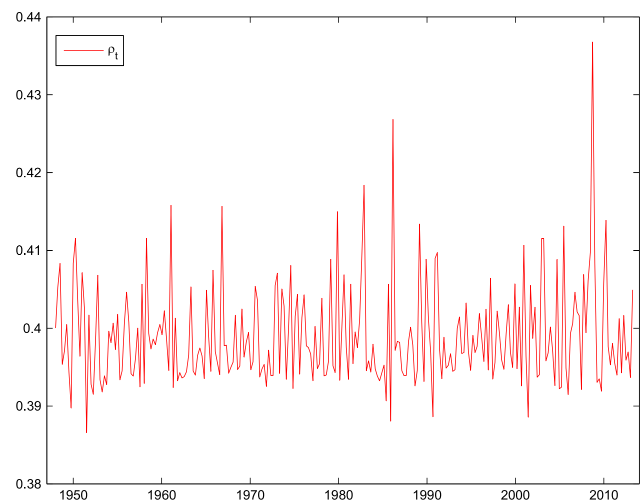

Fig. 2. Time plot of the expected value of $p(\rho_t|y_1, \ldots, y_T), t = 1, \ldots, 261$, obtained from the AR-S-TVP-SVC model.

2.2. The semiparametric TVP-SV model with correlated errors

We relax the parametric assumption for the joint distribution of ε_t and η_t by letting this distribution be unspecified. To this end, we use the Dirichlet process prior which is a powerful tool for modeling unknown distributions. For a detailed description of this prior see Navarro et al. (2006).

The unspecified functional form of $(\varepsilon_t, \eta_t)'$ is given by the following Dirichlet process mixture (DPM) model

$$\begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} | \Lambda_t \sim N(\mathbf{0}, \Lambda_t),$$
$$\Lambda_t \overset{i.i.d}{\sim} G, G|a, G_0 \sim DP(a, G_0), \quad (5)$$
$$G_0 = IW(s_0, S_0), a \sim \mathcal{G}(\underline{c}, \underline{d}),$$

where $\Lambda_t = \begin{pmatrix} \sigma_{y,t}^2 & \sigma_{yh,t} \\ \sigma_{yh,t} & \sigma_{h,t}^2 \end{pmatrix}$. μ_h in expression (3) is set to zero for identification reasons.

Model (5) was first proposed by Jensen and Maheu (2014). According to this model, the conditional distribution of the error vector $(\varepsilon_t, \eta_t)'$ given Λ_t is a bivariate Gaussian with mean zero and random variance–covariance matrix Λ_t. Λ_t is generated from