Economics Letters 155 (2017) 126–130

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Linking individual and collective contests through noise level and sharing rules[☆]

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HIGHLIGHTS

- We use Nitzan's sharing rule to introduce tractable noise in an individual contest.
- The proposed contest satisfies homogeneity of degree zero.
- The proposed contest can be effort or noise equivalent with the Tullock contest.

ARTICLE INFO

Article history:
Received 1 February 2017
Accepted 17 March 2017
Available online 31 March 2017

JEL classification:
C72
D72
D74

Keywords:
Contest
Homogeneity of degree zero
Equivalence

ABSTRACT

We propose the use of Nitzan's (1991) sharing rule in collective contests as a tractable way of modelling individual contests. This proposal (i) tractably introduces noise in Tullock contests when no closed form solution in pure strategies exists, (ii) satisfies the important property of homogeneity of degree zero, (iii) can be effort or noise equivalent to a standard Tullock contest.

1. Introduction

Suppose N players participate in a contest by exerting costly effort to win a prize of common value V. A crucial modelling element in such setups is the contest success function (CSF), f_i, mapping the vector of non-negative efforts to the probability that player $i \in N$ wins the prize (i.e., $f_i : R_+^N \rightarrow [0, 1]$ such that $\sum_{i \in N} f_i(.) = 1$). In the CSF proposed by Tullock (1980),

$$f_i^r(e_1, \ldots, e_N) = \frac{e_i^r}{\sum_{j=1}^{N} e_j^r} \text{ if } \sum_{j=1}^{N} e_j > 0 \text{ and } 1/N \text{ otherwise}$$

(r-**function**)

where $e_i \geq 0$ denotes the effort exerted by player i and $r \geq 0$ determines the level of noise. If $r = 0$ then the noise is maximum and players face a fair lottery. If $r \rightarrow \infty$ then there is no noise and the highest effort wins with certainty (an all-pay auction). Different levels of noise can be introduced for intermediate values.

Although the importance of noise when modelling contests is widely accepted, Tullock's otherwise tractable proposal leads to certain modelling challenges: First, when more than two players with asymmetric costs compete, a closed form solution for the equilibrium in pure strategies exists only if $r = 1$. That is, the introduction of noise in asymmetric multiplayer contests

☆ We gratefully acknowledge the comments by Matthias Dahm, Jörg Franke, Sanghack Lee, Shmuel Nitzan and conference participants in the CBESS Conference on 'Contests: Theory and Evidence' 2016. Any remaining errors are our own. Troumpounis thanks the Spanish Ministry of Economy and Competitiveness for its financial support through grant ECO2015-67171.

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http://dx.doi.org/10.1016/j.econlet.2017.03.020
0165-1765/

becomes intractable. Second, when only two players compete, an equilibrium in pure strategies does not exist for high levels of noise.

Here we propose the allocation of a prize among group members in collective contests as introduced by Nitzan (1991) as a way of addressing these challenges.[1] While guaranteeing tractability, we show that (i) this proposal can be effort or noise equivalent to an r-contest and (ii) several axiomatic and equilibrium properties are similar to Tullock's original proposal. These results differentiate this proposal to a similar approach by Amegashie (2006) as based on Dasgupta and Nti (1998):

$$f_i^{\alpha}(e_1, \dots, e_N) = \frac{e_i + \alpha}{\sum_{j=1}^{N} e_j + N\alpha} \quad (\alpha\text{-}\textbf{function})$$

where $\alpha > 0$ is the introduced "tractable" noise parameter.

From an axiomatic perspective, the r-CSF satisfies all desirable properties of imperfect discrimination, anonymity, monotonicity, homogeneity of degree zero (HD0) and Luce's axiom (Skaperdas, 1996). Achieving noise tractability using the α-function requires the sacrifice of HD0. However, HD0 is desirable for contests where the result should be scale invariant, for instance when it should be irrelevant whether effort expenditures are measured in euros or in dollars or whether effort levels are measured in hours or minutes (see among others Hirshleifer, 2000; Malueg and Yates, 2006; Alcalde and Dahm, 2007; Beviá and Corchón, 2015). HD0 is thus viewed as an essential property whenever outlays are in quantifiable units such as money or time. Our proposal satisfies HD0, while sacrificing Luce's axiom. Hence, researchers and contest designers may choose between the current proposal and the α-CSF as alternative ways of introducing "tractable" noise depending on the importance of HD0 versus Luce's axiom. Moreover, and in contrast to the α-function, we show that our proposal can be effort or noise equivalent to Tullock's original proposal.

2. The λ-contest

Following Nitzan (1991) we define

$$f_i^{\lambda}(e_1, \dots, e_N) = \lambda \frac{e_i}{\sum_{j=1}^{N} e_j} + (1-\lambda)\frac{1}{N} \text{ if } \sum_{j=1}^{N} e_j > 0 \text{ and } 1/N$$

otherwise (λ-**function**).

As discussed, and compared to the α-CSF, it is easy to show that the λ-function satisfies HD0.[2] Assume linear cost functions with $c_i > 0$ denoting the marginal cost of player i, and without loss of generality assume that $c_1 \leq c_2 \leq \dots \leq c_N$, we can define player's i payoff in the λ-contest as[3]:

$$\pi_i^{\lambda} = f_i^{\lambda}(e_1, \dots, e_N)V - c_i e_i. \quad (1)$$

If $\lambda \in [0, 1]$, then the λ-function satisfies the properties of a CSF and is a convex combination of the most common version of a Tullock CSF ($r = 1$) and of a fair lottery ($r = 0$).[4] Parameter λ is associated to the level of noise in the competition and clearly resembles the effect of r in the r-contest. Low values of λ are associated with high levels of noise. Note however that λ need not be restricted in the [0, 1] interval. When $\lambda > 1$ the proposed function f_i^{λ} may take values outside [0, 1] and therefore cannot be interpreted as a CSF representing probabilities. If $\lambda > 1$, then the proposed function allows for transfers among group members or the presence of a compulsory participation fee (Appelbaum and Katz, 1986; Hillman and Riley, 1989). Since this may imply a negative expected payoff for some contestants our setup may violate voluntary participation and hence, as Hillman and Riley (1989) argue, is relevant in situations that involve winners and losers. If one does not want to model such transfers and interpret the λ-function as a CSF then λ needs to be restricted to the [0, 1] interval.

2.1. Equilibrium

The λ-contest as presented by its payoffs in (1) has been previously solved in Hillman and Riley (1989).[5]

Remark 1 (*Hillman and Riley (1989)*). Denote the cost-weighted prize valuations by $V_i = \frac{\lambda V}{c_i}$, there exists a unique equilibrium in pure strategies with player's i effort given by:

$$e_i = \left(1 - \frac{1}{V_i}\frac{(M-1)}{\sum_{j=1}^{M}\frac{1}{V_j}}\right)\frac{(M-1)}{\sum_{j=1}^{M}\frac{1}{V_j}} \quad (2)$$

where M is the number of active players. Player M is the highest marginal cost player for whom the condition $V_M > \frac{(M-2)}{\sum_{i \leq M-1}\frac{1}{V_j}}$ holds.

Using the λ-contest one can solve for the equilibrium efforts in closed form in any asymmetric multiplayer contest. This is not possible in the r-contest. Comparing equilibrium properties across the three (r, α and λ) ways of modelling contests the following results arise:

1. Individual and aggregate effort decreases with the level of noise in both the α and λ-contest. This is different for r-contest where for the two player r-contest comparative statics of aggregate effort with respect to noise levels depend on the degree of asymmetry between the players.
2. Linking the asymmetry with aggregate equilibrium effort in a two-player λ-contest is in line with the standard result of the r-contest (Nti, 1999) and the α-contest since aggregate equilibrium effort decreases in players' asymmetry.
3. In an N-symmetric-players contest adding an additional player increases total effort in the r-contest with an equilibrium in pure strategies and in the λ-contest, while it may decrease total effort in the α-contest.
4. The λ-contest and r-contest cannot sustain an equilibrium where all players are inactive while this may occur in the α-contest.

2.2. Equivalence

Following the definitions by Chowdhury and Sheremeta (2014)[6]:

[1] For a survey on the literature on prize sharing rules see Flamand and Troumpounis (2015).

[2] The λ-function can be obtained from Beviá and Corchón (2015) by setting $\alpha = \frac{1}{N}$, $s = 1$ and $\beta = \frac{N-1}{N}\lambda$ and the HD0 property can be found from there.

[3] For the reasons of interpretability, players' heterogeneity is introduced through cost asymmetries. This is equivalent to asymmetries in terms of valuations (Gradstein, 1995; Corchón, 2007).

[4] Amegashie (2012) proposes a nested two-player contest that ranges from a Tullock to an all-pay auction. A similar structure can also be found in Grossmann (2014) with a nested α-contest.

[5] In their notation, W_i and L_i denote winner and losers' payoffs and the λ-contest is obtained for the particular values $W_i = \frac{\lambda V}{c_i} + (1-\lambda)\frac{V}{c_i N}$ and $L_i = (1-\lambda)\frac{V}{c_i N}$.

[6] In the case of a unique equilibrium (Chowdhury and Sheremeta, 2011), strategic equivalence implies effort equivalence while the opposite need not be true. Moreover, strategic equivalence need not imply payoff equivalence (Chowdhury and Sheremeta, 2014).