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An extension of stochastic volatility model with mixed frequency information

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HIGHLIGHTS

- We extend the basic SV model with mixed frequency information which is referred to as the MF-SV model.
- The MCMC method is discussed to realize the parameter estimation by a mixture approximation model.
- The MF-SV model can significantly identify the time-varying stable component.
- The MF-SV model can improve the in-sample fitting results that outperform the basic SV model.

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ABSTRACT

This paper extends the SV model to the MF-SV model with mixed frequency information. We show the small sample properties with Monte Carlo experiment with MCMC method. The MF-SV model outperforms the basic SV model in the in-sample performance.

1. Introduction

The stochastic volatility (SV) model has been applied to asset pricing (Harvey and Shephard, 1996), particularly derivative pricing (Bansal et al., 2014) since the SV model match discretization of the diffusion process of asset returns (Zheng and Zuo, 2013). In this model, the volatility is driven by a single "unobservable component" factor which ignores the linkage between volatility and other important factors. Instead, numerous studies show that many factors may influence volatility, which leads to the decomposition of volatility components, then specify the component volatility model.

Component volatility models usually decompose the volatility into a long-term and a short-term component (Engle et al., 2013). The long-term component is mainly affected by low-frequency variables (Engle et al., 2013; Zheng and Shang, 2014). In contrast, some high-frequency factors, such as liquidity shocks, contribute to the short-term component. Obviously, different frequency variables need to be considered when constructing a component model. Engle and Rangel (2008) firstly propose a Spline-GARCH model and Engle et al. (2013) further propose the GARCH-MIDAS (mixed-frequency data sampling) model. The MIDAS approach is used to help low frequency variables predict long-term components. Unfortunately, the mixed-frequency volatility model is limited to GARCH family model but none to the SV family model.

This paper decomposes the volatility into a stochastic component and a stable component in the framework of SV model. As

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a result, we specify a mixed frequency SV (MF-SV) model which is a generalized form of the traditional SV model. Similar to the GARCH-MIDAS model, the stable component is characterized by a low-frequency variable via the MIDAS method. Meanwhile, the stochastic component keeps the same form as traditional SV models.

Researchers have developed Markov chain Monte Carlo (MCMC) algorithms for estimating the parameters since the likelihood function for SV models is intractable (see, Nakajima and Omori, 2009). We study the MCMC method to estimate the parameters of MF-SV model. This paper conducts Monte Carlo experiments to evaluate the finite sample performance of the MF-SV model. We also investigate the estimate results of MF-SV model via empirical study.

2. Methodology

2.1. MF-SV model

Let $r_{i,t}$ be the log return on day i during month (quarterly) t. We assume that there are N_t days in period t. Referring to Engle and Rangel (2008), we write the level equation as follows

$$r_{i,t} = \sigma_{i,t}\sqrt{\tau_t}\varepsilon_{i,t} \tag{1}$$

where $\sigma_{i,t}\sqrt{\tau_t}$ represents volatility, which has two components, $\sigma_{i,t}$ and τ_t. The error term $\varepsilon_{i,t}|\psi_{i-1,t} \sim N(0, 1)$.

Engle and Rangel (2008) have point out that the volatility component τ_t is a secular component influenced by low-frequency volatility. The component $\sigma_{i,t}$ is related to short-lived factors. Similarly, we interpret τ_t as a stable component and $\sigma_{i,t}$ as a stochastic component. The level equation can be rewritten as

$$r_{i,t} = \exp\left\{\frac{h_{i,t}}{2} + \frac{\log(\tau_t)}{2}\right\}\varepsilon_{i,t} \tag{2}$$

where $h_{i,t} \equiv \log(\sigma_{i,t}^2)$.

According to the basic SV model, let $y_{i,t} = \log(r_{i,t}^2)$, $\xi_{i,t} = 1.27 + \log(\varepsilon_{i,t}^2)$, then we have

$$y_{i,t} = -1.27 + h_{i,t} + \log(\tau_t) + \xi_{i,t} \tag{3}$$

where $\xi_{i,t}$ follow the $\log(\chi_1^2)$ distribution with one degree of freedom, zero mean, and variance $\pi^2/2$.

The stochastic component can be expressed by the following equation:

$$h_{i,t} = \phi h_{i-1,t} + \eta_{i,t} \tag{4}$$

where $\eta_{i,t} \sim N(0, \sigma^2)$, ξ_i and η_i are independent of each other.

The stable component τ_t is described by some low frequency variable such as realized volatility ($RV_t = \sum_{i=1}^{N_t} r_{i,t}^2$) over a monthly or quarterly horizon (Engle et al., 2013). We set τ_t with log form by smoothing realized volatility in the spirit of the MIDAS regression.

$$\log\tau_t = m + \theta\sum_{p=1}^{P}\varphi_p(\omega_1, \omega_2)RV_{t-p}^n \tag{5}$$

where RV_{t-p}^n is the normalized log low frequency realized volatility.

P is defined as MIDAS lag year which indicates the maximum lag order in Eq. (5). The weighting function $\varphi_p(\omega_1, \omega_2)$ is the "Beta" lag structure.

$$\varphi_p(\omega_1, \omega_2) = \frac{f(p/P, \omega_1, \omega_2)}{\sum_{p=1}^{P} f(p/P, \omega_1, \omega_2)} \tag{6}$$

Table 1
10-component mixture of Gaussian distributions.

w	$\Pr(w = k)$	m_k	ν_k^2
1	0.0061	1.9268	0.1127
2	0.0478	1.3474	0.1779
3	0.1306	0.7350	0.2677
4	0.2067	0.0227	0.4061
5	0.2272	−0.8517	0.6270
6	0.1884	−1.9728	0.9858
7	0.1205	−3.4679	1.5747
8	0.0559	−5.5525	2.5450
9	0.0158	−8.6838	4.1659
10	0.0012	−14.6500	7.3334

where

$$f(x, a, b) = \frac{x^{a-1}(1-x)^{b-1}\Gamma(a+b)}{\Gamma(a) + \Gamma(b)}. \tag{7}$$

Eqs. (3)–(7) form the MF-SV model. Compared with the traditional SV model, we find that when the component τ_t is some constant value, the MF-SV model degenerates into the basic SV model.

2.2. The MCMC method

This paper uses the Bayesian method to realize the parameter estimation. Referring to Nakajima and Omori (2009), we need to approximate the $\log(\chi_1^2)$ distribution by a K-component mixture of Gaussian densities with a mixture approximation model.

Let $y_{i,t}^* = y_{i,t} - \log(\tau_t)$, and then we obtain the following equation

$$y_{i,t}^* = h_{i,t} + \xi_{i,t}^* \tag{8}$$

where $f(\xi_{i,t}^*) = \sum_{k=1}^{K} q_k f_N(\xi_{i,t}^*|m_{i,t}, \nu_{i,t}^2)$ means using K-component mixture of Gaussian densities to approximate the distribution of $\log(\chi_1^2)$, and q_k is the weight of the normal distribution. The mixture approximation model can be written as the linear Gaussian state space model:

$$\begin{pmatrix} y_{i,t}^* \\ h_{i,t+1} \end{pmatrix} = \begin{pmatrix} h_{i,t} \\ \phi h_{i,t} \end{pmatrix} + \begin{pmatrix} \xi_{i,t}^* \\ \eta_{i,t} \end{pmatrix} \tag{9}$$

$$\left\{\begin{pmatrix} \xi_{i,t}^* \\ \eta_{i,t} \end{pmatrix} | s_t = k\right\} \stackrel{L}{=} \begin{pmatrix} m_k + \nu_k^2 z_{i,t}^1 \\ z_{i,t}^2 \end{pmatrix} \tag{10}$$

where both $z_{i,t}^1$ and $z_{i,t}^2$ follow the standard normal distribution.

Referring to Durbin and Koopman (2002), Nakajima and Omori (2009), MCMC method can be realized via the following blocks:

(1) Initialize parameters $\phi, \sigma, m, \theta, \omega$ and state process $\{s_i\}_{i=1}^T$, $\{h_i\}_{i=1}^T$;
(2) Given parameters m, θ, ω, state process $\{s_i\}_{i=1}^T$, $\{h_i\}_{i=1}^T$, data sample $y_{i,t}^*$, sample ϕ, σ with the M–H (Metropolis–Hastings) algorithm. The prior distribution of ϕ is the beta distribution and that of σ is the inverse gamma distribution;
(3) Given ϕ, σ, state process $\{s_i\}_{i=1}^T$, data sample $y_{i,t}^*$, construct the augmented Kalman filter and sample $\{h_i\}_{i=1}^T$ using the simulation smoother;
(4) Given $\phi, \sigma, m, \theta, \omega$, state process $\{h_i\}_{i=1}^T$, data sample $y_{i,t}^*$, sample $\{s_i\}_{i=1}^T$ using a probability mass function;
(5) Given ϕ, σ, state process $\{h_i\}_{i=1}^T$, sample m, θ, ω using the M–H algorithm.