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A minimal sufficient set of procedures in a bargaining model

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HIGHLIGHTS

- We study two-player strategic bargaining games with deterministic procedures.
- We define a class of procedures called normalized procedures.
- Each feasible payoff outcome can be implemented by a normalized procedure.
- Different normalized procedures result in different payoff outcomes.

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ABSTRACT

For a two-player bargaining model, Mao (2016) extends the alternating offers procedure of Rubinstein (1982) to more general procedures and explores which payoff outcomes are feasible, in the sense that they can be supported by some procedures as subgame perfect equilibria. In this paper, we define a special class of procedures called normalized procedures. We show that while the set of normalized procedures can yield all feasible partitions, none of its proper subsets can do so.

1. Introduction

Suppose that two players A and B bargain to divide a cake that is perfectly divisible. In a seminal work, Rubinstein (1982) proves that if the bargaining follows an alternating offers procedure, a unique partition of the cake exists; this can be supported by a subgame perfect equilibrium (SPE). Numerous studies in the literature have examined extensions or applications of the alternating offers bargaining model. For example, see Shaked and Sutton (1984), Binmore (1985), Binmore et al. (1986), Muthoo (1990), Chatterjee et al. (1993), Krishna and Serrano (1996), Watson (1998), In and Serrano (2004) and Ray (2007), among others.[1]

Mao (2016) follows this literature to extend the alternating offers procedure to more general procedures, in which a single player can make proposals in several consecutive rounds. The main result of that study is that the game has a unique SPE outcome and thus extends the analogous result of Rubinstein (1982). The paper also studies the payoff outcomes that can be supported as SPE by choosing some appropriate procedures. For an application, suppose a designer chooses the bargaining procedure. Now, the results in Mao (2016) can help us understand which partition is feasible; that is, the designer can implement it as an SPE. These results are listed in Section 2 for the readers' convenience.

Since different procedures may sometimes lead to the same SPE outcome, the designer can actually implement all feasible partitions by some subset of the set of all procedures. In Section 3, we define a set of special procedures called normalized procedures, which can be constructed by an iterative algorithm. We show in Section 4 that while the designer can implement all feasible partitions using the set of all normalized procedures, a smaller set of procedures may fail to implement some feasible partition. Thus, it is always appropriate for the designer to focus on normalized procedures to achieve a desirable bargaining outcome.

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[1] See Serrano (2008) for a recent survey.

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2. Preliminaries

In this section, we briefly review the notations, model setup, and some of the conclusions of Mao (2016). We refer the readers to that paper for the proofs of all conclusions arrived at in this section.

The player set is $N = \{A, B\}$. Each player $i \in N$ has a constant discount factor $\delta_i \in (0, 1)$. A procedure consists of a (finite or infinite) sequence of A's and B's, and can be denoted by

$$\omega = (\omega_1, \omega_2, \ldots) = (\underbrace{A, \ldots, A}_{n_1}, \underbrace{B, \ldots, B}_{n_2}, \underbrace{A, \ldots, A}_{n_3}, \ldots) \triangleq [n_1, n_2, n_3, \ldots],$$

where $\omega_i = A$ or B is the ith element of the sequence. Without loss of generality, we assume that the first element of each procedure is $\omega_1 = A$. Let $T(\omega)$ be the number of elements ω contains. In particular, $T(\omega) = \infty$ if ω is infinite. Let Ω denote the set of all procedures.

Given δ_A, δ_B, and ω, the bargaining game $G(\omega, \delta_A, \delta_B)$ proceeds as follows. Time is discrete and can be denoted by period $t = 1, 2, \ldots, T(\omega)$. Suppose the game has come to period $t \le T(\omega)$. The proposer in this period is ω_t, who makes an offer d^t from the agreement set $\{(d_A, d_B) \mid d_A, d_B \ge 0,\ d_A + d_B = 1\}$, where d_i is i's share of the cake in the agreement. The other player $i \ne \omega_t$ decides whether to accept or reject this offer. If $d^t = (d_A, d_B)$ is accepted, the game ends and player i's payoff is $u_i(d_i, t) = \delta_i^{t-1} d_i$. If $t \le T(\omega) - 1$ and the offer is rejected, then the game proceeds to the next period $t+1$. If no agreement is ever accepted in all periods $t \le T(\omega)$, both players receive zero payoff.

We solve $G(\omega, \delta_A, \delta_B)$ by subgame perfect equilibrium (SPE).

Theorem 1. *Given $\omega = [n_1, n_2, n_3, n_4, \ldots]$, there exists a unique SPE outcome in which players reach agreement $\big(\theta(\omega), 1 - \theta(\omega)\big)$ without delay, where*

$$\theta(\omega) = 1 - \delta_B^{n_1} + \delta_B^{n_1}\delta_A^{n_2} - \delta_B^{n_1+n_3}\delta_A^{n_2} + \delta_B^{n_1+n_3}\delta_A^{n_2+n_4} - \cdots. \qquad (1)$$

This theorem is an extension of the main theorem of Rubinstein (1982), since, if $\omega = [1, 1, 1, \ldots]$ is an infinite alternating offers procedure, it follows from (1) that $\theta(\omega) = \frac{1-\delta_B}{1-\delta_A\delta_B}$.

More formally, given $\omega = [n_1, n_2, n_3, \ldots]$, we define

$$r(\omega) = \begin{cases} 0, & \text{if } \omega = [n_1] \\ m, & \text{if } \omega = [n_1, n_2, \ldots, n_{m+1}] \\ \infty, & \text{if } \omega \text{ is infinite,} \end{cases}$$

and

$$p(\omega, k) = \begin{cases} 1, & \text{if } k = 0 \\ \delta_B^{n_1}\delta_A^{n_2}\delta_B^{n_3}\cdots\sigma_k^{n_k}, & \text{if } k = 1, 2, \ldots, r(\omega), \end{cases}$$

where $\sigma_k = \delta_A$ if k is even, and $\sigma_k = \delta_B$ if k is odd. We can rewrite (1) as

$$\theta(\omega) = \sum_{k=0}^{r(\omega)} (-1)^k p(\omega, k). \qquad (2)$$

The following two lemmas guarantee that $\theta(\omega)$ is well defined by (2) even when ω is infinite and that $\theta(\omega)$ actually defines a partition of the cake.

Lemma 1. *$\sum_{k=t}^{\infty}(-1)^k p(\omega, k)$ is absolutely convergent for all $t \ge 0$.*

Lemma 2. *For any $\omega \in \Omega$, $1 - \delta_B \le \theta(\omega) \le 1$.*

Furthermore, let $z_r(\omega) = \sum_{k=0}^{r}(-1)^k p(\omega, k)$, $r = 0, 1, \ldots, r(\omega)$; these can be regarded as the SPE partitions of the corresponding truncated procedures of ω. If $\omega = [n_1, n_2, \ldots]$, then $z_0(\omega) = \theta([n_1]) = 1$, $z_1(\omega) = \theta([n_1, n_2]) = 1 - \delta_B^{n_1}$, …, $z_{r(\omega)}(\omega) = \theta(\omega)$. The next lemma implies that the elements in the sequence $\big(z_r(\omega)\big)_{r=0,1,\ldots,r(\omega)}$ are alternately larger and smaller than $\theta(\omega)$.[2]

Lemma 3. *For any $t < s \le r(\omega)$, $z_t(\omega) < z_s(\omega)$ if t is odd, and $z_t(\omega) > z_s(\omega)$ if t is even.*

One possible application of Theorem 1 is as a tool to analyze the influence of the procedure on the bargaining outcome. We are particularly interested in which partitions can be implemented in SPE by choosing appropriate procedures. Let $\Gamma(\delta_A, \delta_B) = \{\theta(\omega) \mid \omega \in \Omega\}$ collect all partitions that are feasible in the sense that they can be supported in SPE by some procedure. Then, the next two theorems show that all partitions are feasible if the players are patient enough, but almost no partitions are feasible if the players are impatient.

Theorem 2. *If $\delta_A + \delta_B \ge 1$, then $\Gamma(\delta_A, \delta_B) = [1 - \delta_B, 1]$.*[3]

Theorem 3. *If $\delta_A + \delta_B < 1$, then the (Lebesgue) measure of $\Gamma(\delta_A, \delta_B)$ is 0.*

Note that we do not consider random procedures in which the proposer is randomly chosen at some period. In fact, when $x \in \Gamma(\delta_A, \delta_B)$, a designer can achieve expected payoffs $(x, 1 - x)$ by designing a one-period random procedure in which (a) player A (or B) has probability x (or $1 - x$) to be the proposer and makes an offer and (b) rejection of this offer leads to the end of the game when both players' payoffs are zero. However, if the designer is risk averse, this random procedure is not as good for the designer as the deterministic procedure, which can implement the same outcome $(x, 1 - x)$ without uncertainty.

3. Normalized bargaining procedures

Given a feasible partition $x \in \Gamma(\delta_A, \delta_B)$, there may exist multiple procedures that result in the same SPE outcome $(x, 1 - x)$. Let $\Omega(x) = \{\omega \in \Omega \mid \theta(\omega) = x\}$ collect all these procedures. For each $\omega \in \Omega(x)$, the elements in the sequence $\big(z_r(\omega)\big)_{r=0,1,\ldots,r(\omega)-1}$ are alternately larger and smaller than x, and they will either reach or converge to x.[4] However, for different $\omega \in \Omega(x)$, $z_r(\omega)$ may approach x at different speeds.

Example 1. Suppose $\delta_A = \frac{3}{5}$, $\delta_B = \frac{2}{3}$, and $x = \frac{1-\delta_B}{1-\delta_A\delta_B} = \frac{5}{9}$. It is easy to verify that both the infinite alternating offers procedure $\omega^1 = [1, 1, \ldots]$ and the finite procedure $\omega^2 = [2, 1]$ are in $\Omega(x)$, i.e. $\theta(\omega^1) = \theta(\omega^2) = x$. A designer whose target outcome is $(x, 1-x)$ might well prefer ω^2 to ω^1, not only because ω^2 is simpler, but also because $z_r(\omega^2)$ approaches x faster than $z_r(\omega^1)$ does, in the sense that $|z_r(\omega^2) - x| \le |z_r(\omega^1) - x|$ for all $r \le \min\{r(\omega^1), r(\omega^2)\}$.

Inspired by the above example, we are interested in a special class of procedures that keep $z_r(\omega)$ as close to $\theta(\omega)$ as possible for each $r \le r(\omega)$. More specifically, for $x \in \Gamma(\delta_A, \delta_B)$, we can construct $\omega \in \Omega(x)$ by using the following algorithm.

We first assume that $\delta_A + \delta_B \ge 1$, and thus $\Gamma(\delta_A, \delta_B) = [1 - \delta_B, 1]$ due to Theorem 2. If $x = 1$, let $\omega = [1]$. Otherwise, we have $1 - \delta_B \le x < 1$. Let n_1 be the integer such that $1 - \delta_B^{n_1} \le x < 1 - \delta_B^{n_1+1}$; that is, n_1 is the maximal n such that $1 - \delta_B^n \le x$. If $x = 1 - \delta_B^{n_1}$, let $\omega = [n_1, 1]$; otherwise, let n_2 be the integer such

[2] Let $s = r(\omega)$ when ω is finite, and let $s \to r(\omega)$ when ω is infinite.

[3] Note that if Ω also contains the procedures that $\omega_1 = B$, then $\Gamma(\delta_A, \delta_B) = [0, 1]$.

[4] That is, either $z_{r(\omega)}(\omega) = x$ when ω is finite, or $\lim_{r\to\infty} z_r(\omega) = x$ when ω is infinite.