Economics Letters 157 (2017) 62–66

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Are rational explosive solutions learnable?

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HIGHLIGHTS

- Study E-stability properties of explosive solutions in models with lagged endogenous variables.
- Explosive solutions are E-stable and strongly E-stable under realistic parameterizations.
- Establish convergence of least squares learning process to explosive solutions.
- In the Cagan model, money supply feedback rule gives rise to a learnable explosive solution.
- Simulations illustrate convergence to the explosive solution for prices.

ARTICLE INFO

Article history:
Received 20 January 2017
Received in revised form 30 March 2017
Accepted 31 May 2017
Available online 2 June 2017

JEL classification:
E52
E32
D83
D84

Keywords:
Money supply
Learnability
Inflation
E-stability

ABSTRACT

It is commonly believed that rational explosive solutions are unstable or fragile under adaptive learning. Contrary to this belief, the paper shows that under realistic parameterizations, rational explosive solutions are both E-stable and strongly E-stable in a class of models with lagged endogenous variables. It also establishes the convergence of least squares learning process to explosive solutions. Taking a simple Cagan model of inflation as an application, the paper shows that money supply feedback rule gives rise to a rational explosive solution for prices which is learnable in real time. This provides a new potential explanation for historical high inflation. Finally, E-stability results for non-MSV explosive solutions are provided.

1. Introduction

It is commonly believed that rational explosive solutions are unstable or fragile under adaptive learning; see Marcet and Sargent (1989a), Evans (1989), Evans and Honkapohja (1992, 2001). Subsequent work usually overlooks the Expectational-Stability (E-stability) properties of explosive solutions; exceptions include Branch and Evans, 2011, Evans and McGough, 2015. Existing literature on the E-stability of explosive solutions typically considers models without lagged endogenous variables.

In linear stochastic economic models with lagged endogenous variables, the paper firstly shows that under realistic parameterizations, the minimum state variable (MSV) explosive solution is both Expectationally-stable (E-stable) and strongly E-stable. It then establishes the convergence of least squares learning process to explosive solutions when agents learn about the growth rate of the explosive variable. As an application, the paper studies the Cagan model of inflation with a money supply feedback rule. It is shown that this rule gives rise to a rational explosive solution for price levels which is learnable. Numerical simulations illustrate the convergence of real-time learning process to the explosive solution. Finally, we provide E-stability results for MSV explosive solutions under alternative parameterizations and for non-MSV explosive solutions.

2. Model

We consider the following class of models with one expectational lead and one lag of endogenous variables studied in Evans

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http://dx.doi.org/10.1016/j.econlet.2017.05.038
0165-1765/

and Honkapohja (2011, henceforth EH) p. 201–204

$$y_t = \alpha + \beta E_t y_{t+1} + \delta y_{t-1} + \kappa w_t + \nu_t, \tag{1}$$

$$w_t = \mu + \rho w_{t-1} + e_t, \tag{2}$$

where $|\rho| < 1$. w_t is an exogenous AR(1) process. ν_t is an *i.i.d* process with mean zero and constant variance.[1] Examples of models taking this form are the Lucas–Prescott model of investment under uncertainty and the Cagan model of inflation with money supply feedback rules.

To solve for MSV rational expectations (RE) equilibria, we assume that agents' PLM is

$$y_t = a + by_{t-1} + cw_t + \eta_t, \tag{3}$$

where η_t are *i.i.d.* regression errors. Calculating conditional expectations $E_t y_{t+1}$ and substituting the expectations into (1) yield the actual law of motion (ALM)

$$y_t = T_1(a, b, c) + T_2(a, b, c)\, y_{t-1} + T_3(a, b, c)\, w_t + \frac{\nu_t}{1-\beta b}. \tag{4}$$

The T-map which maps the coefficients in the PLM to the coefficients in the ALM is $T_1(a, b, c) = \frac{\alpha+\beta(a+c\mu)}{1-\beta b}$, $T_2(a, b, c) = \frac{\delta}{1-\beta b}$, and $T_3(a, b, c) = \frac{\kappa+\beta c\rho}{1-\beta b}$. The RE solutions satisfy $T_1(\bar{a}, \bar{b}, \bar{c}) = \bar{a}$, $T_2(\bar{a}, \bar{b}, \bar{c}) = \bar{b}$, $T_3(\bar{a}, \bar{b}, \bar{c}) = \bar{c}$. The model generally has two solutions where $\bar{b} = \frac{1\pm\sqrt{1-4\beta\delta}}{2\beta}$. Let $\bar{b}_+$ and $\bar{b}_-$ denote the two solutions.

3. Stability and convergence results

This section shows that under realistic parameterizations, the RE explosive solution of the model is both E-stable and strongly E-stable and establishes the convergence of least squares learning process to the explosive solution.

3.1. E-stability and strong E-stability of the explosive solution

Proposition 8.3 in EH provided the E-stability condition of the RE solutions[2]

$$\frac{\beta}{1-\beta\bar{b}} < 1 \tag{5}$$

$$\text{and} \quad \frac{\delta\beta}{\left(1-\beta\bar{b}\right)^2} < 1. \tag{6}$$

While EH focus on the non-explosive solutions, we instead study explosive solutions. We consider the following set of parameters:

$$0 < \beta < 1, \qquad \delta < 0. \tag{7}$$

Note $\bar{b}_+ > \frac{1}{\beta} > 1$ is an explosive solution.

Proposition 1. *The MSV RE explosive solution $\bar{b}_+$ is E-stable if condition (7) holds.*

Given (7), $\frac{\delta\beta}{(1-\beta\bar{b}_+)^2} < 0$ and hence (6) will be satisfied. Note given $\beta\bar{b}_+ > 1$, we have $\frac{\beta}{1-\beta\bar{b}_+} < 0$ and hence (5) holds. Fig. 1 plots the T-map $T_2(b) = \frac{\delta}{1-\beta b}$ when $\beta = 0.99$, $\delta = -0.02$, $\alpha = 0$ and $\kappa = 0$; it is the parameterization of the numerical simulation

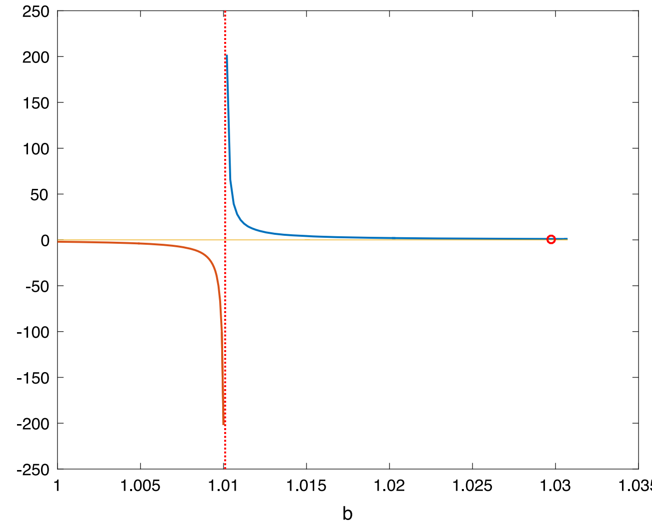

Fig. 1. T-map: $T_2(b) = \frac{\delta}{1-\beta b}$.

in Section 4. The vertical dashed line is $b = \beta^{-1}$. The circled point is the $\bar{b}_+$ solution. In the interval (β^{-1}, ∞), the T-map is a hyperbola and decreasing function in coefficient b. So the $\bar{b}_+$ solution is E-stable.[3]

The following proposition proves that the $\bar{b}_+$ solution is strongly E-stable and the strong E-stability condition is identical to the E-stability condition.

Proposition 2. *The MSV RE explosive solution $\bar{b}_+$ is strongly E-stable if condition (7) holds.*

Proof. Suppose agents' PLM is $y_t = a + by_{t-1} + cw_t + \sum_{i=2}^{p} d_{i-2}y_{t-i} + \sum_{i=1}^{q-1} h_{i-1}w_{t-i} + \eta_t$. We assume that the PLM is over-parameterized relative to the MSV solution and hence at least one extra lag of dependent variables or exogenous variable is added as regressors. Conditional expectations are $E_t y_{t+1} = a + by_t + c(\mu + \rho w_t) + \sum_{i=1}^{p-1} d_{i-1}y_{t-i} + \sum_{i=0}^{q-2} h_i w_{t-i}$. Substituting the expectations into model (1) yields the ALM $y_t = \frac{\alpha+\beta(a+c\mu)}{1-\beta b} + \frac{\delta+\beta d_0}{1-\beta b} y_{t-1} + \frac{\kappa+\beta c\rho+\beta h_0}{1-\beta b} w_t + \frac{\beta\sum_{i=2}^{p-1} d_{i-1}y_{t-i}}{1-\beta b} + \frac{\beta\sum_{i=0}^{q-2} h_i w_{t-i}}{1-\beta b} + \frac{1}{1-\beta b}\nu_t$. Under RE, $\bar{d}_0 = \bar{d}_1 = \cdots = \bar{d}_{p-2} = \bar{h}_0 = \bar{h}_1 = \cdots = \bar{h}_{q-2} = 0$. The convergence of coefficients $\left(d_1, \ldots, d_{p-2}, h_1, \ldots, h_{q-2}\right)$ in the PLM are irrelevant for the E-stability analysis because they are mapped into zeros in the ALM. In addition, because $d_0 = h_0 = 0$ under RE, we get that the strong E-stability condition is identical to the E-stability condition. ■

3.2. Convergence of least squares learning to explosive solution

In the $\bar{b}_+$ explosive solution, dependent variables y_t depend on the constant regressor 1, y_{t-1} and w_t. y_t grow over time but the constant regressor 1 and w_t will become negligible asymptotically relative to the explosive dependent variable. Agents may chase the trend and learn about the growth rate of y_t.[4] Suppose agents

[1] Models with only one shock (i.e., either ν_t or w_t) are nested in (1)–(2). The E-stability results later do not depend on the assumption of having two shocks.

[2] This proposition also contains $\frac{\beta\rho}{1-\beta\bar{b}} < 1$ as the E-stability condition. Note if $\frac{\beta}{1-\beta\bar{b}} < 1$ holds, then $\frac{\beta\rho}{1-\beta\bar{b}} < 1$ also holds, given that $|\rho| < 1$. So $\frac{\beta\rho}{1-\beta\bar{b}} < 1$ is omitted here.

[3] Note the T-map is a decreasing function in the interval $\left(-\infty, \beta^{-1}\right)$, so the stationary MSV solution is also E-stable.

[4] Adam et al. (2012), Kuang and Mitra (2016) develop learning models where agents learn about the (trend) growth rates. These models can generate large fluctuations in housing markets and the business cycle and are consistent with important features of observed macroeconomic expectations.