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How to determine exchange rates under risk neutrality: A note[☆]

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ABSTRACT

The goal of this paper is to determine the exchange rates consistent with an equilibrium in the international assets and goods markets. We present a wealth model of a two-country economy where financial assets and goods are traded. We consider the case where the agents are risk neutral, a very common assumption in finance in order to have explicit solutions for prices, and, in particular, in international finance for exchange rates using the non-null Pareto optima. We show that the Pareto optima in the international assets and goods markets are found to coincide with the net trade allocations. More notably, under a no-arbitrage condition in the assets markets, we can define an exchange rates system for which PPP holds. We provide conditions to have a non-null Pareto optimum to compute the exchange rates. We give an example with a non-null Pareto optimum associated with the determination of the exchange rate.

1. Introduction

The objective of this paper is to determine the exchange rates that are consistent with an equilibrium in the international assets and goods markets. For this purpose, we consider a pure exchange economy with financial assets and international good trade. Under the assumption of risk neutrality, we compute the exchange rates by using the Pareto optima. Most of the papers on finance rest on the assumption of risk neutrality because it gives the possibility to have explicit solutions for the prices at the equilibrium. In international finance, this assumption permits to get an explicit solution for the exchange rates. Our main result is the following. Under a no-arbitrage condition in the financial assets markets, we are able to calculate the exchange rates by using the Pareto optima that coincide, under risk neutrality, with the net trade. We provide conditions to have a non-null Pareto optimum in the determination of the exchange rates. We give also an example where one of these conditions is satisfied. Actually, under risk-neutrality, the exchange rates no longer depend on the Pareto allocations. This property does not hold in presence of risk aversion. In Remark 3, we briefly show that, under risk aversion, the exchange rates depend on the Pareto allocations. Since these allocations are difficult to determine, the computation of the exchange rates turns out to be intractable.

We also show that, when the financial assets markets are complete, the assets are not redundant and no arbitrage condition holds, then, any Pareto optimum and its associated prices clear the trade balance.

Our note is organized into seven sections. Notations and fundamentals are introduced in Sections 2–5. The no-arbitrage conditions in the financial markets are considered in Section 6 . Section 7 bridges risk neutrality, Pareto optimality and the exchange rates determination stating our main results. Section 8 concludes.

2. Model

We focus on a pure exchange economy where financial assets and goods are traded in international markets. We consider a two-period exchange economy introduced by Hart (1974) but with many countries. In the first period, agents trade financial assets to diversify their portfolios and maximize a linear utility function (under risk neutrality). In the second period, they exchange goods spending their initial endowments and the gains from financial investments. They are allowed to exchange goods across the borders, contrary to Dumas (1992). Security returns and goods are valued in domestic currencies. Financial assets are traded in the first period and goods are consumed in the second. The representative agent

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of country $i \in \{1, \ldots, I\}$ purchases K financial assets in period 0 to smooth consumption in period 1 across S states of nature. Nature provides an endowment in period 1.

3. Notations

Let us introduce a compact notation for asset prices and quantities on the financial side of the economy.

$q \equiv (q_1, \ldots, q_K)$ is a row of financial asset prices where q_k denotes the price of financial asset k given in units of good.

$x \equiv \left(x_k^i\right)$ is the $K \times I$ matrix of portfolios where x_k^i denotes the amount of financial asset k in the portfolio of agent i. Column $x^i \equiv \left(x_1^i, \ldots, x_K^i\right)^T$ is the portfolio of agent i.

$R^i \equiv \left(R_{sk}^i\right)$ is the $S \times K$ matrix of returns where $R_{sk}^i \geq 0$ denotes the return[1] on asset k in the state of nature s. R_s^i is the sth row of the matrix. Returns R_{sk}^i are valued in the currency of country i.

Let us provide now a compact notation for beliefs, prices and quantities on the real side of the economy.

$\pi \equiv \left(\pi_s^i\right)$ is the $I \times S$ matrix of beliefs where π_s^i denotes the belief of agent i about the occurrence of state s. The individual row of beliefs $\pi^i \equiv \left(\pi_1^i, \ldots, \pi_S^i\right)$ lies in the S-unit simplex.

In this paper, since we have many countries with different currencies, it is convenient to take the physical good as numeraire. $p \equiv \left(p_s^i\right)$ is the $I \times S$ matrix of good prices where p_s^i is the quantity of goods we obtain with one unit of currency i in the state of nature s. $p^i \equiv \left(p_1^i, \ldots, p_S^i\right)$ is the ith row of the matrix.

$\tau \equiv \left(\tau_s^i\right)$ is the $I \times S$ matrix of exchange rates where τ_s^i denotes the exchange rate between currencies of country 1 and country i in the state of nature s. $\tau^i \equiv \left(\tau_1^i, \ldots, \tau_S^i\right)$ is the ith row of the matrix. The first row is a vector of units: $\tau_s^1 = 1$ for any s.

$w \equiv \left(w_s^i\right)$ is the $S \times I$ wealth matrix where w_s^i denotes the wealth enjoyed by agent i in the state of nature s . $w^i \equiv \left(w_1^i, \ldots, w_S^i\right)^T$ is the wealth column of agent i. The amount w_s^i is valued in the currency of country i and the utility function of any agent depends on her wealth: $u^i = u^i\left(w_s^i\right)$.

$e \equiv \left(e_s^i\right)$ is the $S \times I$ matrix of endowments where e_s^i denotes the endowment nature provides to agent i in the state s. $e^i \equiv \left(e_1^i, \ldots, e_S^i\right)^T$ is the endowments column of agent i. The endowment e_s^i is valued in the currency of country i.

Notice that prices and beliefs q, R_s^i, τ^i, p^i, π^i are rows, while quantities x^i, w^i, e^i are columns.

Recall that in this paper, the physical good is the numeraire. The individual consumption value is given by $p^i w^i$. Since w_s^i is valued in the currency of country i, p_s^i is the quantity of goods we obtain, in state s, with one unit of currency i and $p_s^i w_s^i$ is the number of goods we obtain with w_s^i. We can therefore aggregate the physical value of wealth $p^i w^i$ over the states.

In the article, $\sum_i$, $\sum_s$, $\sum_k$ will denote unambiguously the explicit sums $\sum_{i=1}^I$, $\sum_{s=1}^S$, $\sum_{k=1}^K$.

4. Assumptions

The first triplet of hypotheses specifies the properties of the returns.

Assumption 1. For any country i and any state s, $\sum_k R_{sk}^i > 0$.

Assumption 2. For any country i and any financial asset k, $\sum_s R_{sk}^i > 0$.

When Assumption 1 fails, there is a country i and a state s where any financial asset k yields $R_{sk}^i = 0$. In this case, the representative agent of country i will enjoy her endowment in the state s.

When Assumption 2 fails, there is an asset k yielding $R_{sk}^i = 0$ in any state of nature s in country i: the representative agent i will refuse to buy this financial asset. The following assumption is stronger and implies Assumption 2.

Assumption 3. For any country i and any portfolio $x^i \neq 0$, the return on portfolio is nonzero: $R^i x^i \neq 0$.

Assumption 3 means that there are no nonzero portfolios with a null return in any state of nature. In other terms, whatever country i we consider, rank $R^i = K$ and the K financial assets are not redundant.[2]

The second triplet specifies the properties of the fundamentals (endowments and preferences).

Assumption 4. Endowments are positive: $e_s^i > 0$ for any agent i and any state s.

Assumption 5. Beliefs are positive: $\pi_s^i > 0$ for any agent i and any state s.

Assumption 5 simply means that any representative agent considers each state as possible.

Eventually, preferences are required to satisfy risk neutrality.

Assumption 6. For any agent i, the utility function is $u^i\left(w_s^i\right) = w_s^i$ for $w_s^i \in \mathbb{R}$.

5. Preferences

The agents' behavior comes down to a saving diversification. In the state s, agents exchange their endowments according to their portfolio:

$$w_s^i = e_s^i + R_s^i x^i. \tag{1}$$

Preferences of agent i are rationalized by a Von Neumann–Morgenstern utility function weighted by subjective probabilities: $\sum_s \pi_s^i w_s^i$, where w_s^i is her wealth. Thus, w_s^i is permitted to become negative in some states of nature and the utility function is defined on the whole space: $w_s^i \in \mathbb{R}$. The portfolio set X^i coincides with $\mathbb{R}^K$.

In the first period, agent i diversifies her portfolio in order to satisfy her welfare under the financial budget constraint:

$$\max_{x^i \in \mathbb{R}^K} \sum_s \pi_s^i \left(e_s^i + R_s^i x^i\right) \tag{2}$$

$$q x^i \leq 0.$$

The right-hand side of the budget constraint is zero because we consider the agents' net purchases. Write $q = (q_1 \ldots, q_K)$. For any k, q_k is the quantity of good required to obtain one unit of asset k.

6. Arbitrage

In economics, arbitrage is the practice of taking advantage of a price difference between two markets.

(a) In finance, arbitrage is possible when the same asset does not trade at the same price (in this paper, in units of goods) in two markets. The following condition is the usual no-arbitrage condition for financial assets markets.

[1] The return is the value of one unit of security in the second period including the dividend. Agents form beliefs about the future and associate with each return the probability of its state of nature.

[2] Market completeness means that the columns of R^i span the whole space $\mathbb{R}^S$ (rank $R^i = S$) and implies that a full insurance is possible. Redundancy of financial assets means that $\dim \ker R^i > 0$, that is $K > \text{rank} R^i$. When capital markets are complete and financial assets are not redundant, we have $K = S = \text{rank} R^i$. In this case, the return matrix is square and invertible.