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Discrete-response state space models with conditional heteroscedasticity: An application to forecasting the federal funds rate target

Stefanos Dimitrakopoulos [a,*], Dipak K. Dey [b]

[a] Department of Accounting, Finance and Economics, Oxford Brookes University, UK
[b] Department of Statistics, University of Connecticut, USA

HIGHLIGHTS

- We propose a discrete-response state space model with conditional heteroscedasticity.
- The proposed model is estimated using MCMC methods.
- The proposed model has better forecast performance than benchmarks that have only constant coefficients or constant variance.

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ABSTRACT

We propose a state space mixed model with stochastic volatility for ordinal-response time series data. For parameter estimation, we design an efficient Markov chain Monte Carlo algorithm. We illustrate our method with an empirical study on the federal funds rate target. The proposed model provides better forecasts than alternative specifications.

1. Introduction

Generalized linear state space (GLSS) models for discrete-response time series observations have been well studied in Bayesian literature (West et al., 1985; Fahrmeir, 1992; Song, 2000; Czado and Song, 2008; Stefanescu et al., 2009; Abanto-Valle and Dey, 2014). This class of models consists of two processes. In the first process, an observation or measurement equation defines the conditional mean of a time series of discrete observations as a nonlinear function (known as the inverse link function) of a sequence of latent state variables. In the second process, a transition or state equation describes the (stationary or non-stationary) dynamic process of the randomly time-varying state variables.

GLSS models can capture, through a time-varying parameter specification, the structural instability which may be present in time series of macro(financial) variables. A second well-known characteristic of (macro)financial time series is conditional heteroscedasticity. Researchers have highlighted the importance of allowing for time-varying conditional variances when analyzing discrete-response time series data (Hausman et al., 1992; Bollerslev et al., 1992; Dueker, 1999). However, the Bayesian literature on GLSS models has assumed homoscedastic errors so far.

In this paper, we extend the Bayesian literature on GLSS models by introducing a new class of models, the generalized nonlinear state space (GNLSS) models. The term "nonlinear" is justified by the presence of conditional heteroscedasticity. In the context of our empirical application we show that by accounting for conditional heteroscedasticity we achieve an increase in the forecast performance of GLSS models.

* Correspondence to: Oxford Brookes University, Department of Accounting, Finance and Economics, Oxford, OX33 1HX, UK.
E-mail address: sdimitrakopoulos@brookes.ac.uk (S. Dimitrakopoulos).

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In particular, we develop methods of Bayesian inference in a state space mixed model with stochastic volatility (SV) (Taylor, 1986) for ordinal-valued time series. The stochastic volatility component accounts for some stylized facts of (macro)financial time series such as volatility clustering, heavy tails and high-peakedness. For the proposed ordinal-response model, the inverse link function is assumed to be a normal cumulative distribution function (c.d.f). The term "mixed" refers to the inclusion of both constant and time-varying coefficients in the model. The parameter transitions are captured by a random walk process.

The proposed model contributes also to the literature on discrete-response time series models with conditional heteroscedasticity (Müller and Czado, 2009; Hsieh and Yang, 2009; Yang and Parwada, 2012; Ahmed, 2016). In the context of our empirical application, we show that by not accounting for time-varying parameters, the forecasting ability of discrete-choice models with conditional heteroscedasticity deteriorates.

Our model entails estimation challenges due to its latent nature, the presence of stochastic volatilities as well as the presence of the latent time-varying parameters. Therefore, we resort to Markov chain Monte Carlo methods and devise an efficient algorithm in order to estimate all parameters of interest.

In terms of our empirical application, our point of departure is the famous model of Hamilton and Jorda (2002) who examined the direction and magnitude of change of the Federal funds rate target in the context of an ordered probit specification. We built upon this model to account for time-varying parameters as well as conditional heteroscedasticity and conduct a forecasting exercise. Forecast evaluation is conducted, using point and density forecasts.

The resulting empirical model is inspired by the paper of Dueker (1999) who highlighted the importance of accounting for conditional heteroscedasticity in modeling discrete changes in the bank prime lending rate and the paper of Huang and Lin (2006) who examined the same issue, using an ordered probit model with time-varying parameters.

2. Econometric set up

Consider the following latent time-varying parameter regression model with stochastic volatility

$$y_t^* = \mathbf{x}_t'\boldsymbol{\beta} + \mathbf{z}_t'\boldsymbol{\alpha}_t + \varepsilon_t,\ \ \varepsilon_t \sim N(0, \exp(h_t)),\ \ t = 1, \ldots, T, \tag{1}$$

$$\boldsymbol{\alpha}_{t+1} = \boldsymbol{\alpha}_t + \mathbf{u}_t,\ \ \mathbf{u}_t \sim N(\mathbf{0}, \Sigma),\ \ t = 0, 1, \ldots, T-1, \tag{2}$$

$$h_t = \mu_h + \phi(h_{t-1} - \mu_h) + \eta_t,\ \ |\phi| < 1,\ \ \eta_t \sim N(0, \sigma_\eta^2). \tag{3}$$

Eq. (1) contains the constant coefficient vector, $\boldsymbol{\beta}$, of dimension $k \times 1$ and time-varying coefficients, $\boldsymbol{\alpha}_t$, of dimension $p \times 1$. The design matrix $\mathbf{x}_t$ includes an intercept. The parameter-driven dynamics follow a random walk process which is given in Eq. (2). This process is initialized with $\boldsymbol{\alpha}_0 = \mathbf{0}$ and $\mathbf{u}_0 \sim N(\mathbf{0}, \Sigma_0)$, where Σ_0 is a known initial state error variance.

In expression (3) time-varying volatility is captured by a stochastic volatility model, where h_t is the log-volatility at time t. The dynamics of h_t is governed by a stationary ($|\phi| < 1$) first-order autoregressive stochastic process with unconditional mean μ_h and unconditional variance $\sigma_\eta^2/(1-\phi^2)$; the parameter ϕ measures the persistence in log-volatilities and σ_η^2 is the variance of shock to the log-volatility. We also assume that both the error terms ε_t and η_t are independent for all t.

The variable y_t^* is latent. Instead, we observe the ordinal response variable y_t, where each y_t takes on any one of the J ordered values in the range $1, \ldots, J$. The unobserved variable y_t^* and the observed variable y_t are connected by

$$y_t = j \ \Leftrightarrow\ \ \zeta_{j-1} < y_t^* \le \zeta_j,\ \ 1 \le j \le J. \tag{4}$$

To ensure a properly defined cumulative distribution function for y_t we assume $\zeta_j > \zeta_{j-1}, \forall j$, with $\zeta_0 = -\infty$ and $\zeta_J = +\infty$.

The model, given by the expressions (1)–(4) is the ordinal-response state space mixed model with stochastic volatility (OSSMM-SV model).

For identification reasons, some restrictions need to be imposed on the model. As a location normalization, we set $\zeta_1 = 0$. As a scale normalization we fix an additional cutpoint, setting $\zeta_{J-1} = 1$ (Chen and Dey, 2000).[1] We also transform the cutpoints as follows

$$\zeta_j^* = \log\left(\frac{\zeta_j - \zeta_{j-1}}{1 - \zeta_j}\right),\ j = 2, \ldots, J-2, \tag{5}$$

with $\boldsymbol{\zeta}^*_{(2,J-2)} = (\zeta_2^*, \ldots, \zeta_{J-2}^*)'$. This reparameterization, due to Chen and Dey (2000) allows for an efficient way of simulating the ζ_j's.

We assume the following independent priors over the set of parameters $(\boldsymbol{\beta}, \Sigma, \boldsymbol{\zeta}^*_{(2,J-2)}, \sigma_\eta^2, \mu_h, \phi)$,

$$\boldsymbol{\beta} \sim N(\boldsymbol{\beta}_0, \mathbf{B}),\ \Sigma \sim IW(\delta, \Delta^{-1}),\ \boldsymbol{\zeta}^*_{(2,J-2)} \sim N(\boldsymbol{\mu}_{\zeta^*}, \Sigma_{\zeta^*}),$$
$$\sigma_\eta^2 \sim \mathcal{IG}(v_a/2, v_\beta/2),\ \mu_h \sim N(\bar{\mu_h}, \bar{\sigma_h}^2),\ (\phi+1)/2 \sim Beta(\phi_a, \phi_\beta),$$

where IW and $\mathcal{IG}$ denote the Inverse-Wishart distribution and the inverse gamma distribution, respectively. The prior on $(\phi+1)/2$ ensures that the prior on ϕ has support on $(-1, 1)$. Furthermore, the reparametrization in (5) allows us to place unrestricted priors over $\boldsymbol{\zeta}^*_{(2,J-2)}$. Therefore, for the transformed cutpoints $\boldsymbol{\zeta}^*_{(2,J-2)}$ we assume a multivariate normal prior.

3. Posterior analysis

3.1. MCMC algorithm

Define

$$\boldsymbol{y} = (y_1, \ldots, y_T),\ \boldsymbol{y^*} = (y_1^*, \ldots, y_T^*),\ \boldsymbol{\alpha} = (\boldsymbol{\alpha}_1, \ldots, \boldsymbol{\alpha}_T),$$
$$\mathbf{h} = (h_1, \ldots, h_T).$$

The likelihood function of the proposed model is given by

$$L = p(\boldsymbol{y}|\boldsymbol{\beta}, \boldsymbol{\alpha}, \boldsymbol{\zeta}_{(2,J-2)}, \mathbf{h}) = \prod_{t=1}^{T}\prod_{j=1}^{J} P(y_t = j|\boldsymbol{\beta}, \boldsymbol{\alpha}_t, \zeta_{j-1}, \zeta_j, h_t)^{1(y_t=j)},$$

where

$$P(y_t = j|\boldsymbol{\beta}, \boldsymbol{\alpha}_t, \zeta_{j-1}, \zeta_j, h_t) = \Phi\left(\frac{\zeta_j - \mathbf{x}_t'\boldsymbol{\beta} - \mathbf{z}_t'\boldsymbol{\alpha}_t}{\exp(h_t/2)}\right) - \Phi\left(\frac{\zeta_{j-1} - \mathbf{x}_t'\boldsymbol{\beta} - \mathbf{z}_t'\boldsymbol{\alpha}_t}{\exp(h_t/2)}\right),$$

with $1(y_t = j)$ being an indicator function that equals one if $y_t = j$ and zero otherwise. Φ is the standard Gaussian c.d.f and $\boldsymbol{\zeta}_{(2,J-2)} = (\zeta_2, \ldots, \zeta_{J-2})'$.

The MCMC scheme for the OSSMM-SV model consists of updating the parameters $(\boldsymbol{\beta}, \Sigma, \boldsymbol{\alpha}, \sigma_\eta^2, \mu_h, \phi, \boldsymbol{\zeta}^*_{(2,J-2)}, \boldsymbol{y^*}, \mathbf{h})$. We sample the state vector $\boldsymbol{\alpha}$, using the precision sampler of Chan and Jeliazkov (2009). To update the volatility vector $\mathbf{h}$ we apply the approach of Chan (2017). We update $\boldsymbol{\zeta}^*_{(2,J-2)}$ and $\boldsymbol{y^*}$ in one block, within an independence Metropolis–Hastings step in order to improve efficiency.

Details of the MCMC algorithm, along with a simulation study, are provided in the Online Appendix A.

[1] For various identification schemes of ordinal-response models see Chen and Khan (2003), Hasegawa (2009) and Müller and Czado (2009).