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Stability and auctions in labor markets with job security[☆]

Hu Fu[a,1], Robert Kleinberg[b,2], Ron Lavi[c,*], Rann Smorodinsky[c]

[a] University of British Columbia, Department of Computer Science, Canada
[b] Computer Science Department, Cornell University, United States
[c] Faculty of Industrial Engineering and Management, Technion – Israel Institute of Technology, Israel

HIGHLIGHTS

- Fu et al. (2016) introduced a stability concept for labor markets with job security.
- We show that their proposed outcomes form Nash equilibria of a natural auction.
- In this auction game, firms compete for workers.
- This result parallels literature on stable outcomes and similar auctions.
- This result also yields new price of anarchy bounds for this auction game.

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ABSTRACT

Fu et al. (2016) introduced a stability concept for labor markets with job security. We show that their proposed outcomes form Nash equilibria of an auction where firms compete for workers. This parallels literature on stable outcomes and similar auctions, and yields new price of anarchy bounds.

1. Introduction

The purpose of this note is to study a connection between a model of labor markets with regulation recently defined and analyzed in Fu et al. (2017) and a model of simultaneous single-item auctions which was originally suggested by Bikhchandani (1999) and which has received significant attention in the last few years in the computer science literature on algorithmic game theory. This connection yields new results in both models. In particular, the results of Fu et al. (2017) on labor markets with regulation imply (via the connection we establish here) new bounds on the price of anarchy and the price of stability of simultaneous second price auctions as well as new guarantees on the existence of pure Nash equilibria in this auction game.

In the classical labor market model due to Kelso and Crawford (1982) there is a set of firms N, a set of workers M, and a production function $v^n : 2^M \to \Re_+$ for every firm $n \in N$, where $v^n(S)$ is the production value of firm n if it hires a subset workers $S \in 2^M$. Three main results of this theory are that gross-substitutability of all production functions is a sufficient condition for the existence of a stable matching of workers and firms, that all stable matchings are efficient, and that in fact gross-substitutability is also a necessary condition for the above two properties to hold (this last property is due to Gul and Stacchetti (1999)).

Bikhchandani (1999) studies a complete-information auction game where firms compete simultaneously for employees by

[☆] The results in this note were presented in the 13th ACM Conference on Electronic Commerce (EC'12), see Fu et al. (2012). Only the abstract (25 lines) was included in the proceedings.

* Corresponding author.
E-mail addresses: fu.hu.thu@gmail.com (H. Fu), rdk@cs.cornell.edu (R. Kleinberg), ronlavi@ie.technion.ac.il (R. Lavi), rann@ie.technion.ac.il (R. Smorodinsky).

[1] This work was done while the author was a Ph.D. student in the Computer Science Department at Cornell University and a postdoc at Microsoft Research New England and California Institute of Technology.

[2] Portions of this note were written while the author was a researcher at Microsoft Research New England.

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proposing salaries, and employees pursue the highest offer. This is known in the literature on combinatorial auctions as a *First Price Item Bidding (FPIB)* auction. Bikhchandani (1999) shows a connection between this auction game and the classic labor market model described above, by showing a correspondence between the set of Nash equilibrium (NE) outcomes of the FPIB game and the set of stable outcomes of a labor market.[3]

In a recent paper (Fu et al., 2017) we modify the classic labor market model and study labor markets with regulation designed to provide employees with job security. To capture job security we have introduced a weaker solution concept termed JS-stability (where JS stands for job security). That previous paper has three main results: it provides sufficient conditions on the structure of the production functions that ensure the existence of JS-stability, it shows that the welfare in any JS-stable outcome is at least half of the optimal welfare, and it describes necessary and sufficient conditions for the existence of efficient JS-stable outcomes. These results therefore provide a mirror image of the three main results of classic labor markets (without job security) described above.

Continuing this thread of thought, our result in this current note parallels the connection between the classic labor market and the FPIB game of Bikhchandani (1999). Specifically, we show a connection between labor markets with job security and the *Second Price Item Bidding (SPIB)* auction game, introduced in Christodoulou et al. (2008). The difference between FPIB and SPIB is that in the latter an employee's salary is determined by the second highest offer and not the first. While Bikhchandani (1999) shows a correspondence between the set of pure NE outcomes of the FPIB auction game and the set of stable matchings of the classic labor market (without regulation), we show here a correspondence between the set of pure NE outcomes of the SPIB auction game and the set of JS-stable outcomes in our model of labor markets with regulation. The two theories of labor markets with and without job security are therefore parallel with respect to this property as well. As mentioned above, as an immediate corollary of this connection we obtain several results regarding existence of pure Nash equilibria in the SPIB auction game, and the price of anarchy and price of stability in this game.

Section 2 gives more details on the model of a labor market with regulation. Section 3 analyzes the connection between the labor market model and the auction model, and describes some corollaries that this connection yields.

2. Labor markets with regulation

The classical labor market model due to Kelso and Crawford (1982) is given by a tuple $(N, M, (v^n)_{n\in N})$, where N denotes the set of firms, M the set of workers and $v^n : 2^M \to \Re_+$ is the production function of firm n in monetary units. We assume throughout that the functions v^n are monotonically increasing and calibrate $v^n(\emptyset) = 0$. A job market *allocation* is a pair (A, s), consisting of an *assignment* $A = \{A^1, \dots, A^N\}$ and a salary vector $s = \{s_m\}_{m\in M}$. An assignment A is a partition of the set N of workers, where A^n is the set of workers employed by firm n. Given an allocation (A, s) the utility of worker m is her salary s_m and the utility of firm n is $\Pi^n(A, s) = v^n(A^n) - \sum_{m\in A^n} s_m$, i.e., the value that the firm obtains from employing the workers in A^n minus the sum of salaries to these workers. The *efficiency level* (or welfare level) of an assignment A is $P_v(A) = \sum_n v^n(A^n)$.[4] The central solution concept in the literature on labor markets is that of stability:

Definition 1. An allocation (A, s) is *individually rational* (IR) if, for all n, $\Pi^n(A; s) \geq 0$ and $s_m \geq 0$ for all $m \in A^n$.

Definition 2. A coalition $\{n, C\}$ is a **blocking** coalition for an allocation (A, s) if and only if there exists a vector of salaries, $\hat{s} \in \Re^C_+$, such that:

1. $\hat{s}_m \geq s_m \;\; \forall k \in N, m \in A^k \cap C$ (workers in C are better-off),
2. $v^n(C) - \sum_{m\in C} \hat{s}_m \geq v^n(A^n) - \sum_{m\in A^n} s_m$ (firm n is better-off),

with at least one of the inequalities being strict. An allocation (A, s) is **stable** if and only if it is IR and there exist no blocking coalitions for it.

Stable outcomes (whenever they exist) can be shown to exhibit maximum efficiency level. However, they are guaranteed to exist only when firms' production functions are gross substitutes (Gul and Stacchetti, 1999). Partly motivated by an attempt to broaden existence of stable outcomes and partly motivated by job security regulations common in many labor markets, the current set of authors considered in Fu et al. (2017) a weaker notion of stability termed *JS-stability*, where JS stands for Job Security:

Definition 3. A coalition $\{n, C\}$ is a **JS-blocking** coalition for an allocation (A, s) if and only if it is a blocking coalition, and additionally $A^n \subset C$. An allocation (A, s) is **JS-stable** if and only if it is IR and there exist no JS-blocking coalitions for it.

The extra requirement $A^n \subset C$ is what captures job security. In other words, a blocking firm can only consider adding new workers and cannot unilaterally dismiss any of its current workers. A detailed discussion of the advantages and disadvantages of this stability concept is given in our previous paper (Fu et al., 2017).

In Fu et al. (2017) we showed that JS-stable outcomes are guaranteed to exist for a class of valuation functions much larger than gross-substitutes. The efficiency loss when relaxing the stability notion to JS-stability was quantified as follows.

Definition 4. A firm's production function v is *fractionally subadditive* on a set $C \subseteq M$, if there exist salaries $s \in \Re^C_+$ such that $\sum_{m\in C} s_m = v(C)$ and $\forall D \subset C$, $\sum_{m\in D} s_m \leq v(D)$. A production function v is fractionally subadditive if it is fractionally subadditive on all subsets of M.

Definition 5. A firm's production function v is *almost fractionally subadditive* (AFS) if:

1. For any $C \subset M$ (excluding $C = M$) v is fractionally subadditive on C, and
2. $v(M) \leq \frac{1}{|M|-1} \sum_{m\in M} v(M \setminus m)$.

AFS strictly contains FS (Fu et al., 2017), and FS significantly expands the class of gross-substitutes valuations (Lehmann et al., 2006). The main results in Fu et al. (2017) can be summarized as follows:

Theorem 1 (*Fu et al., 2017*)**.** *Given any job market $(N, M, (v^n)_{n\in N})$,*

1. *If an allocation (A, s) is JS-stable, and if $\bar{A}$ is an assignment that maximizes the efficiency level, then $P(A) \geq \frac{1}{2} P(\bar{A})$.*
2. *If each v^n is in AFS, then an efficient JS-stable outcome is guaranteed to exist.*

Furthermore, for any $u \notin$ AFS there exist an integer k and production functions $v^1, \dots, v^k \in$ AFS such that no efficient JS-stable outcome exists in the job market with production functions $(u, v^1, \dots, v^k)$.

We refer the reader to Fu et al. (2017) for intuition of the AFS class and the proof of the theorem.

[3] The terminology of Bikhchandani (1999) uses buyers and items instead of firms and workers, but the difference is only semantic, as we explain below.

[4] This model can be easily extended to allow for employees' utility to be firm-dependent and to allow for unemployed workers. For more details the reader is referred to Fu et al. (2017).