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Mixed spatial duopoly, consumers' distribution and efficiency

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HIGHLIGHTS

- We model a mixed spatial duopoly with non-uniform consumers' distributions.
- We prove the existence of equilibrium when the consumers' distribution is logconcave.
- We assess the efficiency properties of the non-cooperative equilibrium.
- Unlike the case of uniform distribution, equilibrium is typically inefficient.
- The inefficiency may increase as preferences become more concentrated.

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ABSTRACT

We solve a mixed spatial duopoly with a generic log-concave consumers' distribution. We show that the sub-game perfect equilibrium in prices and locations exists and is generally inefficient, so that the efficiency in the standard uniform distribution case stands out as an exception. Notable examples show that the inefficiency may increase as the distribution becomes more concentrated.

1. Introduction

In the analysis of mixed duopoly, the mixed spatial duopoly model exhibits a distinctive feature: when the strategic interaction between a public (welfare-maximizing) firm and a private (profit-maximizing) firm is modelled according to a Hotelling-type framework, the market outcome is efficient (Cremer et al., 1991). Under the usual assumptions of quadratic transportation costs, constant production costs, unit demand, full market coverage and uniform consumers' distribution, a public and a private firm competing over prices and locations along a linear city end up choosing the locations at which the transportation costs are minimized and welfare is maximized. This is in sharp contrast with the results of other mixed duopoly set-ups, where under both quantity and price competition the existence of a public firm is not sufficient to guarantee that the market outcome be efficient. Indeed, one of the main normative implications of the efficiency result is that, in contrast to common findings in mixed markets analysis, in mixed spatial duopoly there is no advantage for the government to optimally manipulate the public firm objective function, e.g. through a partial privatization policy (Lu and Poddar, 2007).

The efficiency property has been shown to be robust to the existence of cost differentials (Matsumura and Matsushima, 2004) and to the hypothesis of sequential choice of locations, provided the public firm be the leader (Matsumura and Matsushima, 2003), while it vanishes when the assumption of unit demand is replaced

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with that of price-elastic demand at each location (Kitahara and Matsumura, 2013). In this paper we assess its robustness with respect to a fundamental element of any spatial model, namely the shape of the consumers' distribution. By solving a mixed spatial duopoly for generic log-concave distributions, we are able to show that the market outcome is typically inefficient, and that the well-known efficiency property is strictly related to the distribution being uniform.

In particular, in Section 2 we discuss the solution of a mixed spatial duopoly for a large set of consumers' distributions, and we compare the outcome of the strategic interaction between the public and the private firm with the socially efficient outcome. In Section 3 we provide some examples of this comparison with notable distributions. We conclude in Section 4.

2. Mixed spatial duopoly with non-uniform distribution

We consider a mixed spatial duopoly of the Hotelling type: a private profit-maximizing firm (firm 1) competes in prices and locations along a linear city of unit length with a public welfare-maximizing firm (firm 2). In order to focus on the role of the consumers' distribution, we preserve the following standard hypotheses: (a) firms share the same technology and produce at constant unit costs, normalized to zero; (b) consumers' transportation costs are quadratic in distance; (c) the gross consumer surplus is always greater than the price gross of the transportation cost, so that each consumer buys one unit of the good. We depart from previous analyses by relaxing the hypothesis of uniform distribution. Rather, we assume the following:

Assumption 1. For any location $x \in [0, 1]$, a log-concave density $f(x)$ of consumers is defined with the following properties: (i) $f(x) \geq 0$ for all $x \in [0, 1]$, and $f(x) > 0$ for all $x \in (0, 1)$; (ii) if $f(0) = 0$ then $\lim_{x \to 0^+} f'(x) > 0$.

Given this set-up, in the next subsection we discuss the solution of the two-stage game in prices and locations between the private firm and the public firm. In Section 2.2 we solve for the efficient solution and verify under which conditions the two solutions coincide.

2.1. The non-cooperative equilibrium

Denote with x_1 and x_2 the distances of firms 1 and 2 from the left end point 0, and assume, without any loss of generality, that $x_1 < x_2$.[1] Given quadratic transportation costs, and the prices p_1 ad p_2 set by the firms, the location z of the consumer who is indifferent between patronizing either firm satisfies:

$$p_1 + (z - x_1)^2 = p_2 + (x_2 - z)^2$$

so that

$$z = z(p_1, p_2; x_1, x_2) = \frac{1}{2}\left(\frac{p_2 - p_1}{x_2 - x_1} + x_2 + x_1\right). \tag{1}$$

Accordingly, the demand functions faced by the firms are respectively:

$$D_1 = \int_0^z f(x)\,dx = F(z), \qquad D_2 = \int_z^1 f(x)\,dx = 1 - F(z)$$

where $F : [0, 1] \to [0, 1]$ is the cumulative consumers' distribution. Therefore, the objective function of firm 1 is:

$$\pi_1 = p_1 F(z). \tag{2}$$

Since firm 2 maximizes welfare – the sum of both firms' profits and the consumers' net surplus – its objective collapses to minimizing total transportation costs T:

$$T = \int_0^z (x - x_1)^2 f(x)\,dx + \int_z^1 (x - x_2)^2 f(x)\,dx. \tag{3}$$

The price stage. At the price stage, minimization of (3) by the public firm yields the following FOC:

$$\frac{\partial T}{\partial p_2} = f(z)\,\frac{dz}{dp_2}\left[(z - x_1)^2 - (z - x_2)^2\right] = 0.$$

Since the SOC is verified, this implies that independently of the shape of $f(\cdot)$ the reaction function of firm 2 is:

$$p_2 = p_1. \tag{4}$$

As to the private firm 1, profit maximization requires:

$$\frac{\partial \pi_1}{\partial p_1} = F(z) + p_1 f(z)\,\frac{\partial z}{\partial p_1} = 0$$

which, using (1), boils down to[2]:

$$p_1 = 2\frac{F(z)}{f(z)}(x_2 - x_1). \tag{5}$$

Given (4) and (5), we can now establish the following proposition.

Proposition 1. *For all $f(\cdot)$ satisfying Assumption 1 there exists a unique Nash equilibrium in prices for any pair of locations (x_1, x_2).*

Proof. By total differentiation of (5), the slope of the reaction function of firm 1, $p_1(p_2)$, is

$$\frac{dp_1}{dp_2} = \frac{\left(1 - \frac{F(z)}{f(z)}\frac{f'(z)}{f(z)}\right)}{1 + \left(1 - \frac{F(z)}{f(z)}\frac{f'(z)}{f(z)}\right)} < 1$$

due to the log-concavity of $f(\cdot)$. Assume now that $p_2 = 0$. Since $x_1 < x_2$, there always exists a positive price p_1 which ensures positive profits to firm 1 by attracting customers located near the left end of the linear city. Therefore, the best reaction to $p_2 = 0$ is some $p_1(0) > 0$. Given that along (5) $dp_1/dp_2 < 1$, there exists a unique price $\widehat{p}_2$ such that $p_1(\widehat{p}_2) = \widehat{p}_2$ and both (4) and (5) are verified. ■

Therefore, the Nash equilibrium in prices is:

$$\widehat{p}_1 = \widehat{p}_2 = \frac{2F\left(\frac{x_1 + x_2}{2}\right)}{f\left(\frac{x_1 + x_2}{2}\right)}(x_2 - x_1). \tag{6}$$

The location stage. At the location stage, the public firm minimizes (3) with respect to x_2, and the private firm maximizes (2) with respect to x_1, by taking into account the solution of the price stage—which implies $z = \widehat{z}(x_1, x_2) = (x_1 + x_2)/2$. Therefore, the public firm's FOC at the location stage is:

$$\begin{aligned}\frac{\partial T}{\partial x_2} &= -2\int_{\widehat{z}}^1 (x - x_2) f(x)\,dx \\ &= \mu - \int_0^{\widehat{z}} x f(x)dx - x_2\left[1 - F(\widehat{z})\right] = 0\end{aligned}$$

[1] Notice that we are not imposing any *a priori* boundary on the location of firms. Following Cremer et al. (1991), we assume that firms have different locations. This is confirmed at equilibrium.

[2] As to the SOC for firm 1,

$$\frac{\partial^2 \Pi_1}{\partial p_1^2} = \frac{1}{(x_2 - x_1)}\left(-f(z) + F(z)\frac{f'(z)}{f(z)}\frac{1}{2}\right) < 0$$

since log-concavity implies $Ff' < f^2$.