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A suggestion for constructing a large time-varying conditional covariance matrix

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HIGHLIGHTS

- This paper proposes a new way of estimating large conditional covariance matrices.
- This proposal is easy to implement on standard software such as Eviews.
- We present Monte Carlo simulations demonstrating its consistency in a variety of cases.

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ABSTRACT

The construction of large conditional covariance matrices has posed a problem in the empirical literature because the direct extension of the univariate GARCH model to a multivariate setting produces large numbers of parameters to be estimated as the number of equations rises. A number of procedures have previously aimed to simplify the model and restrict the number of parameters, but these procedures typically involve either invalid or undesirable restrictions. This paper suggests an alternative way forward, based on the GARCH approach, which allows conditional covariance matrices of unlimited size to be constructed. The procedure is computationally straightforward to implement. At no point in the procedure is it necessary to estimate anything other than a univariate GARCH model.

1. Introduction

Finance theory provides many areas where the calculation of a large conditional covariance matrix is called for. Examples include optimal portfolio allocations of the Markowitz type, value at risk calculations for large numbers of assets and capital asset pricing calculations for a large number of assets. It is well-recognized that in empirical work these matrices should be measured by a *conditional* covariance matrix, since this is the true measure of risk, and that, in general, these conditional covariance matrices should be time-varying. The natural econometric analogue used to measure such a time-varying conditional covariance matrix is the GARCH family of models, developed from Engle (1982), and a direct multivariate extension of the GARCH model, developed by Bollerslev (1986). However, the empirical application of multivariate GARCH presents a problem of dimensionality because standard multivariate GARCH models quickly generate very large numbers of parameters as the number of variables in the system increases. For example, consider the following system:

$$\begin{aligned} &Y_t = \beta_t X + \varepsilon_t \quad \varepsilon_t \sim N(0, \Sigma_t) \\ &vech\,(\Sigma_t) = W + Avech\,(\varepsilon_{t-1}\varepsilon'_{t-1}) + Bvech(\Sigma_{t-1}) \end{aligned} \tag{1}$$

where Y is a vector of n endogenous variable, X is a vector of suitable exogenous variables, ε_t is an innovation vector, Σ a conditional variance–covariance matrix, and we have limited the model to a system GARCH (1,1) specification. This model is a direct generalization of the standard univariate GARCH model, but it is intractable for anything other than a very small number of

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Table 1
Monte Carlo results for a constant covariance.

True value	−0.95	−.75	−.5	−.25	0	0.25	0.5	0.75	1
Estimated	−0.98	−.751	−.5	−.26	−0.00005	0.258	0.52	0.8	1

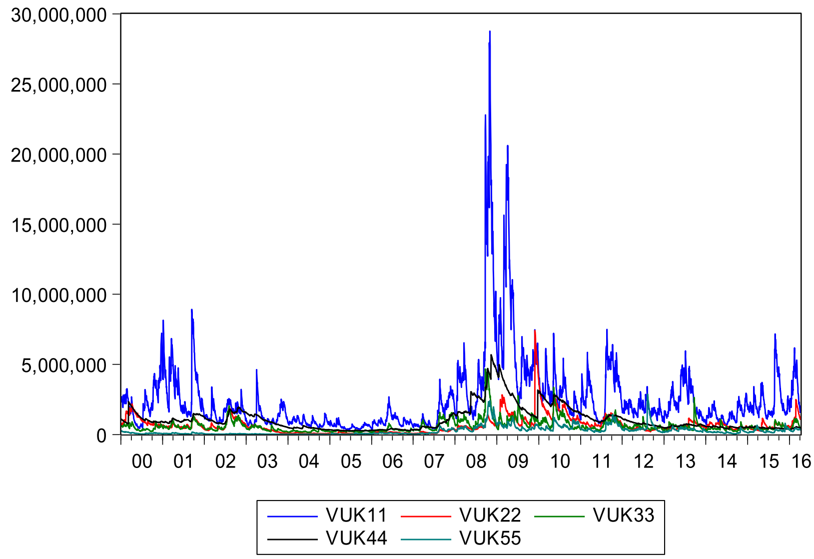

Fig. 1. Conditional variances.

variables. For example, if the number of variables in the system were 5, the model would require estimation of 465 parameters in the W, A and B matrices; the number of parameters grows exponentially with n, the number of variables in the model. There are a number of ways to reduce this problem of dimensionality, but none of them is entirely satisfactory. It is possible to make the A and B matrices diagonal, but this effectively eliminates the interaction between the covariances and severely limits their time-variation. A popular model, based on the work of Baba et al. (unpublished), known as the BEKK model, is given below:

$$\Sigma_t = V'V + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'\Sigma_{t-1}B \tag{2}$$

where A and B are matrices of parameters and V is an upper triangular matrix of parameters.

This model allows fairly complex interactions between the covariances and also ensures that the covariance matrix is positive definite, but it nevertheless involves a large number of parameters as n rises. For a system where $n = 5$, the model contains 75 parameters in the variance equation, and the number again grows rapidly as n rises.

Another alternative would be to use *factor* GARCH models; the assumption here is that there are only a small number of factors underlying the variables being modelled, allowing a much more parsimonious formulation of the model. However, factor GARCH models limit the amount of time-variation in the covariances; also, the assumption of a small number of factors may also be questioned. A further commonly-used approach is the constant conditional correlation model in which the time-varying conditional covariances are parameterized to be proportional to the product of the corresponding conditional standard deviations. This condition, however, restricts precisely the part of the model we are most interested in, and, where this has been tested, this assumption is almost invariably rejected.

There is, therefore, a need for a technique that will allow estimation of large conditional covariance matrices efficiently and without the restrictions imposed by the above methods. In Section 2, we provide such a method based on a variant of this technique recently used to derive a measure of systemic risk for the European banking system (Gibson et al., 2016). Section 3 presents a simple Monte Carlo illustration of the method and performs an example based on some real world data. Section 4 concludes.

2. A suggested approach

Our suggestion rests on a simple relationship between the variance of two variables and the variance of the sum of those two variables. Considering two variables x and y, then

$$\begin{aligned} E(x_t + y_t)^2 &= E[(x_t + y_t)(x_t + y_t)] \\ &= E(x_t^2) + E(y_t^2) + 2E(x_t y_t) \end{aligned} \tag{3}$$

or in conditional variance terms:

$$\mathrm{Var}(x_t + y_t|\Omega_t) = \mathrm{Var}(x_t|\Omega_t) + \mathrm{Var}(y_t|\Omega_t) + 2Cov(x_t y_t|\Omega_t). \tag{4}$$

To estimate the conditional covariance between any two variables, we may estimate a univariate GARCH model for each of the variables and then estimate a further univariate GARCH model for the sum of the variables. Then, we simply calculate the covariance as:

$$\begin{aligned} Cov(x_t y_y|\Omega_t) &= [\mathrm{Var}(x_t + y_t|\Omega_t) \\ &\quad - \mathrm{Var}(x_t|\Omega_t) - \mathrm{Var}(y_t|\Omega_t)]/2. \end{aligned} \tag{5}$$

Specifically, our approach consists of the following steps. (1) We add x and y and we run a GARCH on that sum, obtaining the variance of the sum of x and y. (2) We run a GARCH on x and y separately. Those two steps provide all three terms on the right-hand-side of Eq. (5) so that we have worked-out the covariance matrix. Note that this procedure allows us to derive all of the covariance elements of a large covariance matrix.

If each of the conditional variance estimates is consistent, then our estimate of the covariance will also be a consistent estimate. The theoretical properties of standard GARCH models has been extensively discussed in the literature and the conditions under which they are consistent is well established—see Weiss (1986). The application in (5) is, however, a little unusual as it