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An analytical approach to new Keynesian models under the fiscal theory

Fei Tan

Department of Economics, John Cook School of Business, Saint Louis University, 3674 Lindell Boulevard, St. Louis, MO 63108-3397, USA
Interdisciplinary Center for Social Sciences, Zhejiang University, 38 Zheda Road, Hangzhou, 310027, China

HIGHLIGHTS

- I present a frequency-domain method for solving linear rational expectations models.
- I derive an analytical solution to new Keynesian models under the fiscal theory.
- The solution makes clear the cross-equation restrictions and policy transmission mechanisms.
- The method yields useful by-products which are not easily obtainable using time-domain methods.

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ABSTRACT

This article illustrates a widely applicable frequency-domain methodology to solving multivariate linear rational expectations models. As an example, we solve a prototypical new Keynesian model under the assumption that primary surpluses evolve independently of government liabilities, a regime in which the fiscal theory of the price level is valid. The resulting analytical solution is useful in characterizing the cross-equation restrictions and illustrating the complex interaction between the fiscal theory and price rigidity. We also highlight some useful by-products of such method which are not easily obtainable for more sophisticated models using time-domain methods.

1. Introduction

This article builds on the seminal work, most notably of Hansen and Sargent (1980) and Whiteman (1983), that developed analytical approaches of integrating dynamic economic theories with econometric methods for the purpose of formulating and interpreting economic time series. We show that the frequency-domain methodology of Tan and Walker (2015) to solving linear rational expectations models, who generalized its predecessors to the multivariate setting, is widely applicable for solving well-known dynamic macroeconomic models. In particular, we walk the reader through the details in applying such method and highlight some useful by-products which are not easily obtainable for sophisticated models using time-domain methods.

As an example, we solve a prototypical new Keynesian model of the kind presented in Woodford (2003) and Galí (2008). This has the advantage of keeping the illustration simple and concrete, but it should be emphasized that the techniques we describe are of wide applicability in more general settings, e.g. models with a maturity structure, which we leave for future research. We derive an analytical solution to a linearized version of the model under the assumption that primary surpluses evolve independently of government liabilities, a regime in which the fiscal theory of the price level is valid (Leeper, 1991; Woodford, 1995; Cochrane, 1998; Davig and Leeper, 2006; Sims, 2013). This solution is useful in characterizing the cross-equation restrictions and illustrating the complex interaction between the fiscal theory and price rigidity. It also presents a new way of testing the validity of this theory. An equivalent derivation using time-domain methods, as well as an extensive study of the fiscal theory, can be found in Leeper and Leith (2016).

E-mail address: tanf@slu.edu.

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2. A prototypical new Keynesian model

The model's essential elements include: a representative household and a continuum of firms, each producing a differentiated good; only a fraction of firms can reset their prices each period; a cashless economy with one-period nominal bonds B_t that sell at price $1/R_t$, where R_t is the monetary policy instrument; lump-sum taxation and zero government spending so that consumption equals output, $c_t = y_t$; a monetary authority and a fiscal authority.

2.1. Linearized system

Let $\hat{x} \equiv \ln(x_t) - \ln(x^*)$ denote the log-deviation of a variable x_t from its steady state x^*. It is straightforward to show that a linear approximation to the model's equilibrium conditions leads to the following equations. First, the household's optimizing behavior, when imposed by the goods market clearing condition, implies

$$\hat{y}_t = \mathbb{E}_t\hat{y}_{t+1} - \sigma(\hat{R}_t - \mathbb{E}_t\hat{\pi}_{t+1}) \tag{2.1}$$

where $\sigma > 0$ is the elasticity of intertemporal substitution, $\pi_t = P_t/P_{t-1}$ is the inflation between periods $t-1$ and t, and $\mathbb{E}_t$ represents mathematical expectation given information available at time t. The firm's optimal price-setting behavior reduces to

$$\hat{\pi}_t = \beta\mathbb{E}_t\hat{\pi}_{t+1} + \kappa\hat{y}_t \tag{2.2}$$

where $0 < \beta < 1$ is the discount factor and $\kappa > 0$ is the slope of the so-called new Keynesian Phillips curve.

Next, the monetary authority follows an interest rate feedback rule that reacts to deviations of inflation from its steady state

$$\hat{R}_t = \alpha\hat{\pi}_t + \theta_t \tag{2.3}$$

where θ_t is an exogenous policy shock.[1] In addition, the fiscal authority sets an exogenous primary surplus process, s_t, that evolves independently of government liabilities. This profligate fiscal policy requires that the monetary policy adjust nominal interest rate only weakly to inflation deviations, i.e. $0 \leqslant \alpha < 1$ (Leeper, 1991). We assume that $(\theta_t, \hat{s}_t)$ is jointly a white noise, normally distributed with mean zero and covariance matrix Σ.

Lastly, any policy choice must satisfy the flow government budget constraint, $\frac{1}{R_t}\frac{B_t}{P_t} + s_t = \frac{B_{t-1}}{P_t}$, which is linearized as

$$\hat{b}_t = \hat{R}_t + \beta^{-1}(\hat{b}_{t-1} - \hat{\pi}_t) - (\beta^{-1} - 1)\hat{s}_t \tag{2.4}$$

where $b_t = B_t/P_t$ is the real debt at the end of period t. Note that the real value of outstanding debt at the beginning of period t, $\hat{b}_{t-1} - \hat{\pi}_t$, is determined in equilibrium at time t. (2.1)–(2.4) constitute a system of expectational difference equations in the variables $\{\hat{y}_t, \hat{\pi}_t, \hat{R}_t, \hat{b}_t\}$, which fully characterizes the model dynamics under the fiscal theory.

2.2. Analytical solution

To simplify the exhibition, we substitute the monetary policy rule (2.3) into (2.1) and (2.4) and rewrite the resulting system in the following form

$$\Bigg[\underbrace{\begin{pmatrix}1 & \sigma & 0\\ 0 & \beta & 0\\ 0 & 0 & 0\end{pmatrix}}_{\Gamma_{-1}} L^{-1} + \underbrace{\begin{pmatrix}-1 & -\alpha\sigma & 0\\ \kappa & -1 & 0\\ 0 & \beta^{-1}-\alpha & 1\end{pmatrix}}_{\Gamma_0} L^0 + \underbrace{\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\beta^{-1}\end{pmatrix}}_{\Gamma_1} L\Bigg]$$
$$\times \underbrace{\begin{pmatrix}\hat{y}_t\\ \hat{\pi}_t\\ \hat{b}_t\end{pmatrix}}_{\hat{x}_t} = \underbrace{\begin{pmatrix}\sigma & 0\\ 0 & 0\\ 1 & 1-\beta^{-1}\end{pmatrix}}_{\Psi_0} L^0 \underbrace{\begin{pmatrix}\theta_t\\ \hat{s}_t\end{pmatrix}}_{\varepsilon_t} + \underbrace{\begin{pmatrix}1 & \sigma & 0\\ 0 & \beta & 0\\ 0 & 0 & 0\end{pmatrix}}_{\Gamma_{-1}} \underbrace{\begin{pmatrix}\eta^y_{t+1}\\ \eta^\pi_{t+1}\\ \eta^b_{t+1}\end{pmatrix}}_{\eta_{t+1}} \tag{2.5}$$

where L is the lag operator: $L^k\hat{x}_t = \hat{x}_{t-k}$, $\{\Gamma_{-1}, \Gamma_0, \Gamma_1, \Psi_0\}$ are matrix coefficients, and η_{t+1} is a vector of endogenous forecasting errors defined as $\eta_{t+1} = \hat{x}_{t+1} - \mathbb{E}_t\hat{x}_{t+1}$ so that $\mathbb{E}_t\eta_{t+1} = 0$.

Suppose a solution $\hat{x}_t = [\hat{y}_t, \hat{\pi}_t, \hat{b}_t]'$ to (2.5) is of the form

$$\hat{x}_t = \sum_{k=0}^{\infty} C_k\varepsilon_{t-k} \equiv C(L)\varepsilon_t \tag{2.6}$$

where $\varepsilon_t = [\theta_t, \hat{s}_t]'$, $\hat{x}_t$ is taken to be covariance stationary, and $C(L)$ is a polynomial in the lag operator. Note that such moving average representation of the solution is very useful because it also leads to the impulse response function—the coefficient $C_k(i,j)$ measures exactly the response of $\hat{x}_{t+k}(i)$ to a shock $\varepsilon_t(j)$. In what follows, we walk the reader through the key steps in deriving the content of $C(\cdot)$.

Step 1: transform the time-domain system (2.5) into its equivalent frequency-domain representation. To this end, we evaluate the forecasting errors $\eta_{t+1} = [\eta^y_{t+1}, \eta^\pi_{t+1}, \eta^b_{t+1}]'$ using (2.6) and the Wiener–Kolmogorov optimal prediction formula

$$\eta_{t+1} = \left\{C(L)L^{-1} - \left[\frac{C(L)}{L}\right]_+\right\}\varepsilon_t = C_0L^{-1}\varepsilon_t \tag{2.7}$$

where $[\cdot]_+$ is the annihilation operator that ignores negative powers of L. An implicit assumption underlying (2.7) is that the history of monetary and fiscal shocks are perfectly observed up to period t. Define $\Gamma(L) = \Gamma_{-1}L^{-1} + \Gamma_0 + \Gamma_1L$ and substitute (2.6) and (2.7) into (2.5)

$$\Gamma(L)C(L)\varepsilon_t = (\Psi_0 + \Gamma_{-1}C_0L^{-1})\varepsilon_t$$

which must hold for all realizations of ε_t. Therefore, the coefficient matrices are related by the z-transform identities

$$z\Gamma(z)C(z) = z\Psi_0 + \Gamma_{-1}C_0$$

where z is a complex variable. In solving for $C(z)$, ideally one would multiply both sides by $(z\Gamma(z))^{-1}$, but $C(z)$ needs to have only non-negative powers of z by (2.6) and be analytic inside the unit circle so that its coefficients are square-summable by covariance stationarity. This requirement can be examined by a careful decomposition of $z\Gamma(z)$ in the next step.

Step 2: apply the Smith canonical decomposition to the polynomial matrix $z\Gamma(z)$[2]

$$z\Gamma(z) = \underbrace{U(z)^{-1}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & z(z-\beta)(z-\lambda_-)\end{pmatrix}}_{S(z)} \underbrace{\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & z-\lambda_+\end{pmatrix} V(z)^{-1}}_{T(z)}$$

which factorizes all roots inside the unit circle from those outside and collects them in the diagonal polynomial matrix $S(z)$.

[1] For analytical clarity, we assume that the monetary authority does not respond to output deviations.

[2] The Smith decomposition is available in MAPLE or MATLAB's Symbolic Toolbox.