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# Semiparametric Bayesian inference for time-varying parameter regression models with stochastic volatility

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## HIGHLIGHTS

- A semiparametric stochastic volatility model with time-varying parameters is considered.
- An efficient Markov Chain Monte Carlo estimation algorithm is proposed.
- The proposed model is applied to inflation modelling.
- The proposed model performs better than alternative specifications.

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ABSTRACT

We develop a Bayesian semiparametric method to estimate a time-varying parameter regression model with stochastic volatility, where both the error distributions of the observations and parameter-driven dynamics are unspecified. We illustrate our methodology with an application to inflation.


## 1. Introduction

A vast literature has demonstrated the gains from allowing for time-varying parameters in stochastic volatility models (TVP-SV models), when analysing (macro) financial data (Primiceri, 2005; Cogley and Sargent, 2005; Stock and Watson, 2007; D'Agostino et al., 2013; Clark and Ravazzolo, 2015). Due to the presence of the stochastic volatility component the likelihood function for this class of models is intractable. As a result, researchers have developed Markov chain Monte Carlo (MCMC) algorithms for estimating the model parameters (see, for example, Nakajima, 2011).

In this paper, we consider two semiparametric extensions of the TVP-SV model, utilizing a popular Bayesian prior for modelling unknown distributions, the Dirichlet process (DP) prior (Ferguson, 1973). We first use this prior to model in a flexible way the distribution of the dependent variable's innovation and second, to consider wider class of the distribution of the time-varying parameter's innovation. The resulting semiparametric TVP-SV model is referred to as the S-TVP-SV model. To estimate the model parameters and the unknown distributions, we propose an efficient MCMC algorithm.

The first semiparametric extension has already been applied in the context of standard stochastic volatility models (Jensen and Maheu, 2010; Delatola and Griffin, 2011). The second semiparametric extension is novel and constitutes our main contribution to the Bayesian semiparametric literature on TVP-SV models.

The motivation behind the S-TVP-SV model stems from the empirical literature on inflation modelling. Recently, evidence has

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been found of non-normality in modelling inflation persistence, leading to increased interest in non-Gaussian (fat-tailed) distributions for modelling inflation dynamics (Lanne and Saikkonen, 2011; Lanne et al., 2012; Chiu et al., 2014; Lanne, 2015). Our point of departure is an autoregressive version of the unobserved components with stochastic volatility (UC-SV) model, proposed by Stock and Watson (2007). Stock and Watson (2007) considered a UC-SV model that decomposed inflation into a trend and a transitory component and assumed fat-tailed error distributions for the observation and state equations to control for outliers.

In this paper, we generalize the approach of Stock and Watson (2007) to account for shocks that may not be symmetrically distributed, as economic systems may react differently in recessions and expansionary periods. Furthermore, if there are different regimes operating within the sample period, a fat-tailed distribution may be inadequate to capture this data characteristic. In our proposed model, each of the unconditional error distributions for the observations and the parameter-driven dynamics is allowed to follow an infinite mixture of normals.

## 2. Econometric set up

### 2.1. The TVP-SV model

Consider the following TVP-SV model

$$y_t = \mathbf{x}_t'\boldsymbol{\beta} + \mathbf{z}_t'\boldsymbol{\alpha}_t + \varepsilon_t, \quad \varepsilon_t \sim N(\mu, \exp(h_t)),\ t = 1, \ldots, T, \tag{1}$$

$$\boldsymbol{\alpha}_{t+1} = \boldsymbol{\alpha}_t + \mathbf{u}_t, \quad \mathbf{u}_t \sim N(\mathbf{0}, \boldsymbol{\Sigma}),\ t = 0, 1, \ldots, T-1, \tag{2}$$

$$h_{t+1} = \mu_h + \phi h_t + \eta_t, \quad |\phi| < 1,\ \eta_t \sim N(0, \sigma_\eta^2). \tag{3}$$

Eq. (1) contains two types of coefficients: the constant coefficient vector, $\boldsymbol{\beta}$, of dimension $k \times 1$ and time-varying coefficients, $\boldsymbol{\alpha}_t$, of dimension $p \times 1$. $\mathbf{x}_t$ and $\mathbf{z}_t$ are the design matrices which do not include an intercept and $h_t$ is the log-volatility at time $t$.

Eq. (2) is a random walk process which is initialized with $\boldsymbol{\alpha}_0 = \mathbf{0}$ and $\mathbf{u}_0 \sim N(\mathbf{0}, \boldsymbol{\Sigma}_0)$, where $N(\cdot, \cdot)$ denotes the normal distribution with the initial state error variance $\boldsymbol{\Sigma}_0$ being known.

The error terms $\varepsilon_t$ and $\eta_t$ are assumed to be independent[1] for all $t$. The error term $\varepsilon_t$ follows a normal distribution with mean $\mu$ and time-varying variance $\sigma_t^2 = \exp(h_t)$. The dynamics of the log-volatility $h_t = \log(\sigma_t^2)$ are described by Eq. (3) which is a stationary ($|\phi| < 1$) first-order autoregressive process. This process is initialized with $h_1 \sim N(\mu_h/(1-\phi), \sigma_\eta^2/(1-\phi^2))$. The parameter $\phi$ is the persistence volatility that measures the degree of autocorrelation in $h_t$, and $\sigma_\eta$ is the standard deviation of the shock to log-volatility.

We assume the following priors over the set of parameters $(\boldsymbol{\beta}, \sigma_\eta^2, \boldsymbol{\Sigma}, \mu_h, \mu)$,

$$\boldsymbol{\beta} \sim N(\boldsymbol{\beta}_0, \mathbf{B}), \qquad \sigma_\eta^2 \sim \mathcal{IG}(v_a/2, v_\beta/2), \qquad \boldsymbol{\Sigma} \sim IW(\delta, \Delta^{-1}),$$
$$\mu_h \sim N(\bar{\mu_h}, \bar{\sigma_h}^2), \qquad \mu \sim N(\bar{\mu}, \bar{\sigma}^2),$$

where $IW$ and $\mathcal{IG}$ denote the Inverse-Wishart distribution and the inverse gamma distribution, respectively. To guarantee that the persistence parameter $\phi$ satisfies the stationarity restriction, we assume $(\phi + 1)/2 \sim Beta(\phi_a, \phi_\beta)$.

### 2.2. Two semiparametric extensions

The advantage of Dirichlet process modelling results from its theoretical properties, one of which is the clustering property. A detailed exposition of the statistical properties of the DP prior is given, among others, by Ghosal (2010).

The error term $\varepsilon_t$, is assumed to have an unspecified functional form based on the following Dirichlet process mixture (DPM) model

$$\begin{aligned}
&\varepsilon_t|\vartheta_t, \qquad h_t \sim N(\mu_t, \lambda_t^2 \exp(h_t)), \quad \vartheta_t = (\mu_t, \lambda_t^2),\ t = 1\ldots, T,\\
&\vartheta_t \overset{i.i.d.}{\sim} G,\\
&G|a, G_0 \sim DP(a, G_0),\\
&G_0 = N(\mu_t; \mu_0, \tau_0\lambda_t^2)\mathcal{IG}\left(\lambda_t^2; \frac{e_0}{2}, \frac{f_0}{2}\right),\\
&a \sim \mathcal{G}(\underline{c}, \underline{d}),
\end{aligned} \tag{4}$$

where $\mu_h$ in the stochastic volatility equation is set to zero for identification reasons. The unspecified functional form of the distribution of $\varepsilon_t$, given in (4), was first proposed by Jensen and Maheu (2010).

According to specification (4), the conditional distribution of $\varepsilon_t$ given $h_t$ and $\vartheta_t$ is Gaussian with mean $\mu_t$ and variance $\lambda_t^2 \exp(h_t)$. The $\vartheta_t = (\mu_t, \lambda_t^2)$ is generated from an unknown distribution $G$. For the prior base distribution $G_0$ we assume a conjugate normal-inverse gamma, $N(\mu_t; \mu_0, \tau_0\lambda_t^2)\mathcal{IG}(\lambda_t^2; \frac{e_0}{2}, \frac{f_0}{2})$. A gamma prior distribution $\mathcal{G}(\underline{c}, \underline{d})$ is placed upon $a$, which is the precision parameter (positive scalar). As $a$ tends to infinity $G$ converges pointwise to $G_0$.

One can show that the unconditional distribution of $\varepsilon_t$ follows an infinite mixture model with time-varying means and variances. So our DPM model is able to capture asymmetries and multiple modes that may characterize the data.

Furthermore, to capture the uncertainty about the distribution of $\mathbf{u}_t$, we impose on it the following novel flexible structure,

$$\begin{aligned}
&\mathbf{u}_t|\omega_t, \qquad \boldsymbol{\Sigma} \sim N(0, \omega_t^{-1}\boldsymbol{\Sigma}), \quad t = 1, \ldots, T-1,\\
&\omega_t \overset{i.i.d.}{\sim} G_\omega,\\
&G_\omega|a_\omega, G_{0\omega} \sim DP\left(a_\omega, G_{0\omega} = \mathcal{G}\left(\frac{e_\omega}{2}, \frac{e_\omega}{2}\right)\right),\\
&a_\omega \sim \mathcal{G}(\underline{c}_\omega, \underline{d}_\omega).
\end{aligned} \tag{5}$$

The positive scale parameter $\omega_t$ in (5) comes from an unknown discrete distribution $G_\omega$. The Dirichlet process prior in (5) is defined by the parameter $a_\omega$ and the base gamma distribution $G_{0\omega}$. As the precision parameter $a_\omega$ tends to infinity, $G_\omega$ converges pointwise to $G_{0\omega}$. In this case, the unconditional distribution of $\mathbf{u}_t$ is a multivariate Student-$t$ distribution with $e_\omega$ degrees of freedom and as $e_\omega$ increases the error distribution mimics the Normal distribution. For small values of $a_\omega$ the unconditional distribution of $\mathbf{u}_t$ is a finite mixture of multivariate normals, each of which has the same mean. Therefore, our semiparametric approach for the distribution of $\mathbf{u}_t$ can capture the potential clustering in the mixing scalar parameter of the innovation's covariance matrix.

The TVP-SV model combined with the DPM models of (4) and (5) produces the semiparametric TVP-SV model (S-TVP-SV model).

## 3. Posterior analysis

### 3.1. The MCMC algorithm for the S-TVP-SV model

Define

$$\boldsymbol{y} = (y_1, \ldots, y_T), \qquad \boldsymbol{\alpha} = (\boldsymbol{\alpha}_1, \ldots, \boldsymbol{\alpha}_T), \qquad \mathbf{h} = (h_1, \ldots, h_T),$$
$$\boldsymbol{\theta} = (\vartheta_1, \ldots, \vartheta_T), \qquad \vartheta_t = (\mu_t, \lambda_t^2), \qquad \boldsymbol{\omega} = (\omega_1, \ldots, \omega_{T-1}).$$

Our MCMC scheme for the semiparametric model consists of two parts. In part I, we update the parameters $(\boldsymbol{\beta}, \boldsymbol{\Sigma}, \sigma_\eta^2, \boldsymbol{\alpha}, \mathbf{h}, \phi)$ and

[1] In the context of stochastic volatility models, Jensen and Maheu (2014) assumed that the errors $\varepsilon_t$ and $\eta_t$ are correlated and modelled them nonparametrically, using DP priors.