Economics Letters 150 (2017) 63–66

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

# Voluntary public good provision with private information using order statistics

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ARTICLE INFO

*Article history:*
Received 24 October 2016
Accepted 4 November 2016
Available online 16 November 2016

*JEL classification:*
D61
D82
H41

*Keywords:*
Weaker-link
Better-shot
Privately-provided public good
Complementarity
Private information

ABSTRACT

We study the voluntary provision of a public good with private information when inputs are not perfectly substitutable. Modeling the production function as a mixture of order statistics of individual efforts, we bridge the extreme best-shot and weakest-link technologies, passing through summation, in a tractable framework. In contrast with existing predictions, increasing complementarity results in increased public good provision, if the marginal cost of effort rises sufficiently fast.



## 1. Introduction

We study the interplay of private information and input complementarity for the voluntary provision of public goods. Input complementarity is known to be relevant to public goods; indeed, Hirshleifer (1983) lists several examples including dike maintenance and provision of antimissile batteries. The same public good framework is used for team production, e.g., Ray et al. (2007), and it is fundamental to group competition models.[1] Even monetary donations may not be perfectly substitutable, as demonstrated by the insistence of the recent Sanders campaign on small donations. Input complementarity for weaker-link and better-shot public goods appears in Cornes (1993), Cornes and Sandler (1996), Arce and Sandler (2001), and Cornes and Hartley (2007), among others.

The dominant framework in these complete-information treatments aggregates individual efforts into a team one through a CES production function. Varying elasticity, one bridges best-shot and weakest-link technologies, passing through summation, to explore an interesting economic tradeoff. On the one hand, making individual efforts more complementary reduces free riding. On the other hand, the technology itself becomes worse. Existing results show either no effect of the elasticity parameter for homogeneous agents (Cornes, 1993), or favor this second effect. In particular, in their contest models with technology spanning the range between weakest-link and perfect substitutes, Kolmar and Rommeswinkel (2013) and Cubel and Sanchez-Pages (2014) show that increasing complementarity within a single heterogeneous group reduces its winning probability. Much remains unknown, especially between perfect-substitutes and best-shot technologies. And these issues are unexplored with private information, except for the polar best-shot, weakest-link, and perfect-substitutes technologies (Barbieri and Malueg, 2014, 2015, forthcoming).

Rather than analyzing a CES production function, with its intractable convolutions of exponentiated random variables, our production function is a weighted average of order statistics. This is in the spirit of Arce and Sandler (2001): "For weaker-link public goods, the smallest contribution has the largest marginal influence on utility, followed by the second smallest contribution, and so on. The reverse holds for better-shot".[2] We characterize the unique symmetric equilibrium in differentiable, strictly increasing strategies, providing sufficient conditions for its existence. Focusing on the weighted average of smallest

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[1] Recent contributions include Gould and Winter (2009), Kolmar and Rommeswinkel (2013) and Cubel and Sanchez-Pages (2014), who provide many real-world examples.

[2] See Arce and Sandler (2001, p. 496). They apply their formalization to full-information matrix games examples.

http://dx.doi.org/10.1016/j.econlet.2016.11.005

and largest order statistics, we analyze the effects of increasing complementarity of efforts. In contrast with the contest results previously reported, increasing complementarity raises public good provision if the marginal cost of effort increases very rapidly.

## 2. Model

There are $n \geq 2$ players, indexed by $i$, any of whom exerts effort $x_i \in \mathbb{R}^+$, independently and simultaneously, at cost $c(x_i)$. Assume $c' > 0, c'' > 0, c'(0) = \underline{c} \geq 0$, and $\lim_{x_i \uparrow \infty} c'(x_i) = \infty$. For each unit of the public good $G$, let $i$'s valuation be $v_i$, which is private information, so $i$'s effort is function of his/her valuation: $x_i = g_i(v_i)$. Valuations are i.i.d. continuous random variables with cdf $F$ and pdf $f$ on $[\underline{v}, \bar{v}]$, where $\underline{v} \geq 0$. Player $i$'s realized utility is $v_i \times G - c(x_i)$.

We describe the production technology for $G$ using order statistics. Within efforts $x_1, \ldots, x_n$, denote the $k$th order statistic as $x_{(k)}$, so $x_{(1)} \leq x_{(2)} \leq \cdots \leq x_{(n)}$. The public good level is $G = \sum_{l=1}^{n} p_{(l)} x_{(l)}$, where $p_{(1)}, \ldots, p_{(n)}$ are non-negative weights summing to 1. If $p_{(l)} = 1/n \; \forall l \in \{1, \ldots, n\}$, then, essentially, we have the "summation" production function. The "weakest link" results if $p_{(1)} = 1$ and $p_{(l)} = 0 \; \forall l \in \{2, \ldots, n\}$. If $p_{(n)} = 1$ and $p_{(l)} = 0 \; \forall l \in \{1, \ldots, n-1\}$, then we recover the "best shot". The simplest form of this technology, $G = (1-p_{(n)})x_{(1)} + p_{(n)}x_{(n)}$, allows for different marginal rates of substitution between the largest and smallest contributions by varying $p_{(n)}$.

## 3. Analysis

We characterize symmetric Bayes–Nash equilibrium when the common equilibrium strategy $g$ displays $g' > 0$. For simplicity, begin with $p_{(k)} = 1$ and consider agent $i$ that behaves as if having valuation $v_i'$, while all other agents use $g(\cdot)$. Partition the possible realizations of $v_{-i} \equiv (v_1, \ldots, v_{i-1}, v_{i+1}, \ldots, v_n)$ into three regions according to $m$, the number of individual realizations below $v_i'$:

1. $m < k-1$. Equivalently, the $(k-1)$th lowest valuation in $v_{-i}$ – denoted with $v_{i^*}$ – is above $v_i'$. Therefore, the $k$th lowest valuation in $(v_i', v_{-i})$ is $v_{i^*}$. Since $g' > 0$, $v_{i^*}$ determines $G$: $G = x_{i^*} = g(v_{i^*})$.
2. $m = k-1$. Here, $G = g(v_i')$.
3. $m > k-1$. Equivalently, the $k$th lowest valuation in $v_{-i}$ – denoted with $v_{j^*}$ – is below $v_i'$. Here, $G = g(v_{j^*})$.

Standard results yield the density of $v_{i^*}$ as $f_{v_{i^*}}(y) = (n-1)f(y)\binom{n-2}{k-2}F(y)^{k-2}(1-F(y))^{n-k}$. (Intuitively, there are $n-1$ ways for any one valuation to equal $y$ while exactly $k-2$ out of $n-2$ valuations are below $y$.) The probability of $k-1$ out of $n-1$ values being below $y$ is $P_k(y) = \binom{n-1}{k-1}F(y)^{k-1}(1-F(y))^{n-k}$, and the density of $v_{j^*}$ is $f_{v_{j^*}}(y) = (n-1)f(y)\binom{n-2}{k-1}F(y)^{k-1}(1-F(y))^{n-1-k}$. Therefore, $i$ expects the public good quantity to be

$$G_k^E(v_i') = \int_{v_i'}^{\bar{v}} g(y) f_{v_{i^*}}(y)\, dy + P_k(v_i')g(v_i') + \int_{\underline{v}}^{v_i'} g(y) f_{v_{j^*}}(y)\, dy;$$

so

$$\frac{dG_k^E(v_i')}{dv_i'} = g(v_i')\underbrace{\left[-f_{v_{i^*}}(v_i') + P_k'(v_i') + f_{v_{j^*}}(v_i')\right]}_{=0} + P_k(v_i')g'(v_i') = P_k(v_i')g'(v_i'), \tag{1}$$

because

$$\begin{aligned} P_k'(y) &= \binom{n-1}{k-1} f(y)\left[(k-1)(F(y))^{k-2}(1-F(y))^{n-k} - (n-k)(F(y))^{k-1}(1-F(y))^{n-k-1}\right] \\ &= (n-1)f(y)\left[\binom{n-2}{k-2}(F(y))^{k-2}(1-F(y))^{n-k} - \binom{n-2}{k-1}(F(y))^{k-1}(1-F(y))^{n-k-1}\right] \\ &= f_{v_{i^*}}(y) - f_{v_{j^*}}(y). \end{aligned} \tag{2}$$

More generally for $p_{(k)} \neq 1$, a player with valuation $v$ acting as $v'$ has expected payoff

$$V(v', v) = -c(g(v')) + v \sum_k p_{(k)} G_k^E(v'),$$

so

$$\frac{\partial V(v', v)}{\partial v'} = -c'(g(v'))g'(v') + v \sum_k p_{(k)} \frac{dG_k^E(v')}{dv'} = \underbrace{g'(v')\left[-c'(g(v')) + v \sum_k p_{(k)} P_k(v')\right]}_{\text{by (1)}}. \tag{3}$$

The truth-telling first-order condition is

$$0 = \left.\frac{\partial V(v', v)}{\partial v'}\right|_{v'=v} = g'(v)\left[-c'(g(v)) + v \sum_k p_{(k)} P_k(v)\right]. \tag{4}$$

Letting

$$\beta(v) \equiv v \sum_k p_{(k)} P_k(v) = v\left[p_{(1)}(1-F(v))^{n-1} + \sum_{k=2}^{n-1} p_{(k)} P_k(v) + p_{(n)}(F(v))^{n-1}\right], \tag{5}$$

if $g' > 0$, (4) gives

$$c'(g(v)) = \beta(v), \tag{6}$$

balancing marginal cost and benefit of effort. Since $c'(g(v))$ in (6) must increase in $v$, we have this equilibrium characterization:

**Proposition 1.** *If $\underline{c} \leq \beta(\underline{v})$ and $\beta'(v) > 0$, then the only symmetric equilibrium strategy $g$ such that $g' > 0$ is*

$$g(v) = (c')^{-1}(\beta(v)). \tag{7}$$

**Proof.** Necessity and uniqueness follow by (4) and (6), and $\underline{c} \leq \beta(\underline{v})$. Sufficiency follows by (3) and (6), since

$$\frac{\partial V(v', v)}{\partial v'} = g'(v')\left[-c'(g(v')) + v \sum_k p_{(k)} P_k(v')\right] = g'(v')\left[-\beta(v') + \frac{v}{v'}\beta(v')\right] = \frac{g'(v')\beta(v')}{v'}(v - v'),$$

so $V(v', v)$ is strictly quasi-concave in $v' \in [\underline{v}, \bar{v}]$. The marginal benefit of effort $x > g(\bar{v})$ is $v p_{(n)}$, but

$$v p_{(n)} \leq \bar{v} p_{(n)} \underbrace{=}_{\text{by (5)}} \beta(\bar{v}) \underbrace{=}_{\text{by (6)}} c'(g(\bar{v})) \underbrace{<}_{\text{by } c''>0} c'(x), \tag{8}$$

making deviations above $g(\bar{v})$ unprofitable. Deviations below $g(\underline{v})$ are similarly discouraged, thus completing the proof of equilibrium. □

Lemma 1 gives a sufficient condition for $\beta'(v) > 0$; intuitively, inputs should be sufficiently substitutable.

**Lemma 1.** *If $p_{(1)} \leq p_{(2)} \leq \cdots \leq p_{(n)}$, then $\beta'(v) > 0$.*