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On the disclosure of ticket sales in charitable lotteries

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HIGHLIGHTS

- We study the effect of disclosure of ticket sales on revenue in charitable lotteries.
- A policy of disclosing ticket sales in a fundraising lottery increases revenue.
- The optimal timing of disclosure is when half of the players have purchased tickets.

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ABSTRACT

We show that a policy of disclosing the ticket sales during a fundraising lottery raises total revenue when there are more than two bettors. The optimal timing of the disclosure is when about half of the players have purchased lottery tickets.

1. Introduction

Charitable lotteries constitute a significant source of funding for the provision of public goods and services both at a national and a local level. Proceeds from lottery ticket sales are used to fund health, education and environmental protection initiatives as well as sports, arts, and national heritage programs. According to the North American Association of State and Provincial Lotteries, in 2014 lottery ticket sales in the United States exceeded $70 billion. That is, in 2014 Americans spent more on lottery tickets than on sports events, books, video games, movies, and music combined (see Isidore, 2015; Thompson, 2015). Given the high revenues generated by lottery ticket sales, even small changes in the way lotteries are organized and operated can have a substantial impact on funding for charitable causes.

The effectiveness of lotteries as a means of raising funds for public goods was discussed by Morgan (2000) who showed that a lottery generates more funds than a voluntary contribution scheme. The recent literature has examined various ways to further enhance revenues in fundraising lotteries whereby most efforts have been focused on the use of alternative ticket pricing schemes. In a two-player framework, Franke and Leininger (2014) show that, when donors are budget constrained and heterogeneous, it is optimal to bias the lottery in favor of a specific player, and such a biased lottery design is able to generate the efficient amount of public good provision. Damianov (2015) demonstrates that discounts on lottery tickets raise total revenue when players are sufficiently heterogeneous in the way they value the prize.

In this paper we explore an alternative, easily implementable way of enhancing revenues in fundraising lotteries. We study the impact of disclosing the number of lottery tickets purchased at a given stage of the fundraising event on subsequent sales, on initial sales, and ultimately on total revenue. We show that such a disclosure policy increases total revenue in lotteries with more than two bettors whereby the highest amount of funds is raised when the disclosure occurs once half of the bettors have bought tickets.

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2. The baseline model

We consider a lottery with $N \geq 2$ players who arrive at a fundraising event and decide on the number lottery tickets to purchase. The value of the lottery prize is denoted by v and the revenue generated by ticket sales is used to finance a public good. The per capita return from the public good is denoted by α, whereby $0 \leq \alpha < 1$. In the baseline model without disclosure of ticket sales, the expected payoff of bettor i is given by

$$u(x_i, x_{-i}) = \frac{x_i}{x_i + \sum_{j \neq i} x_j} \cdot v - x_i + \alpha \cdot \left(x_i + \sum_{j \neq i} x_j \right),$$

where $x_i \geq 0$ is the amount spent on lottery tickets by bettor $i = 1, 2, \ldots, N$ and x_{-i} is the vector of amounts spent by the other players.[1] It is easy to demonstrate that the so defined lottery game is isomorphic (i.e. has the same equilibria) to a game without a public good component but with an adjusted prize of $V = \frac{v}{1-\alpha}$. Without loss of generality we can normalize the value of the adjusted prize to unity ($V = 1$) and obtain the following alternative representation for the expected payoff of player i

$$U(x_i, x_{-i}) = \frac{x_i}{x_i + \sum_{j \neq i} x_j} - x_i.$$

In the baseline model of no disclosure of ticket sales bettors move simultaneously. The considered game is thus equivalent to a classical Tullock contest. It is well known that in Nash equilibrium each bettor spends the amount

$$x_i^* = \frac{N-1}{N^2}$$

and total revenue equals

$$Y^* = \frac{N-1}{N}.$$

3. Disclosure of ticket sales

We assume now that the number of tickets sold is disclosed once the first n players who arrive have bought lottery tickets. After this information is revealed, the remaining $k = N - n$ players also purchase lottery tickets upon their arrival. That is, the disclosure policy creates a sequential structure, and we explore here the behavior of the first movers and the second movers in a symmetric subgame perfect Nash equilibrium.

In the following analysis we first establish a relationship between ticket purchases of players in the second stage of the game, y and total revenue Y in a symmetric subgame perfect equilibrium (Lemma 1). Then we derive relationships between ticket purchases in the first stage of the game x and total revenue Y in equilibrium (Lemma 2). These steps allow us to derive our main result, which establishes how equilibrium revenue Y in the lottery depends upon the number of first movers n and the number of second movers k in the game (Proposition 1). We show that the lottery revenue when n players move first and k players move second is equal to the revenue when k players move first and n players move second (Corollary 1). We further demonstrate that the policy of disclosing ticket sales raises more revenue compared to a non-disclosure policy when there are three or more bettors (Corollary 2). That is, the well-known revenue equivalence between the Stackelberg and Cournot rent-seeking game with two bettors (see, e.g. Dixit, 1987; Linster, 1993) breaks down once the number of players is increased. Finally, we prove that the maximum revenue is obtained when about half of the players buy lottery tickets before ticket sales are disclosed (Corollary 3).

Let all the first movers spend the amount $x \geq 0$ and all the second movers, except for player j, spend the amount $y \geq 0$. Player j chooses y_j so as to maximize

$$\frac{y_j}{n\,x + (k-1)\,y + y_j} - y_j.$$

The first order condition is given by

$$\frac{n\,x + (k-1)\,y}{(n\,x + (k-1)\,y + y_j)^2} = 1. \tag{1}$$

In a symmetric equilibrium we require that $y_j = y$, and as a response to the first movers playing x in the first stage, using Eq. (1) we obtain that the second movers play $y(x)$ implicitly given by the equation

$$n\,x + (k-1)\,y = (n\,x + k\,y)^2. \tag{2}$$

Total revenue in the lottery corresponds to the sum of tickets sold in the first stage and tickets sold in the second stage and is given by $Y = n\,x + k\,y$. With these preliminaries we establish our first result.

Lemma 1 (*Second Movers*). *In a symmetric subgame perfect equilibrium, tickets purchased by a player in the second stage of the game y and total revenue Y satisfy the following relationship*

$$y = Y - Y^2.$$

Proof. As total revenue is $Y = n\,x + k\,y$ from Eq. (2) we obtain $Y - y = Y^2$. □

We now rearrange Eq. (2) and define

$$\varphi(x, y) := n\,x + (k-1)\,y - (n\,x + k\,y)^2 \equiv 0.$$

Next, we calculate the derivative $y'(x)$ as the ratio

$$\begin{aligned} y'(x) &= - \frac{\partial\,\varphi(x,y)}{\partial\,x} \Big/ \frac{\partial\,\varphi(x,y)}{\partial\,y} \\ &= \frac{n\,[\,2\,(n\,x + k\,y) - 1\,]}{k\,[\,1 - 2\,(n\,x + k\,y)\,] - 1} = \frac{n\,(2Y - 1)}{k - 2\,k\,Y - 1}. \end{aligned}$$

The reaction of a follower resulting from a change in the strategy of one of the first movers at the point $x_i = x$ while the other first movers play x is hence given by

$$y'(x_i) = \frac{y'(x)}{n} = \frac{2\,Y - 1}{k - 2\,k\,Y - 1}. \tag{3}$$

We can now move to the analysis of the first stage of the game. The payoff of player i in a subgame perfect equilibrium when all other first movers play x and all the second movers react with $y(x_i)$ is given by

$$\frac{x_i}{x_i + (n-1)\,x + k\,y(x_i)} - x_i.$$

For the first order condition we obtain

$$\frac{(n-1)\,x + k\,y(x_i) - x_i\,k\,y'(x_i)}{[\,x_i + (n-1)\,x + k\,y(x_i)\,]^2} = 1. \tag{4}$$

With these preliminaries we establish our second result.

[1] In the case in which no player buys lottery tickets, i.e. $x_i + \sum_{j \neq i} x_j = 0$, we assume that the prize is not awarded and the expected payoff of each player is zero.