Economics Letters 151 (2017) 19–22

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

The Amiti–Weinstein estimator: An equivalence result

Joris Tielens[a,*], Jan Van Hove[a,b]

[a] KU Leuven, Department of Economics, Belgium
[b] KBC Group, Chief Economist Research Department, Belgium

HIGHLIGHTS

- Amiti and Weinstein (2013) propose a framework to estimate bank supply shocks.
- We prove equivalence of the proposed estimator with weighted least squares (WLS).
- The proof relies on the Frisch and Waugh theorem.
- This insight is useful as most statistical software packages have a WLS routine.
- We argue that the estimator can be used in other research venues as well.

ARTICLE INFO

Article history:
Received 12 August 2016
Received in revised form 25 November 2016
Accepted 27 November 2016
Available online 30 November 2016

JEL classification:
E51
C58

Keywords:
Credit demand shocks
Credit supply shocks
Weighted least squares

ABSTRACT

Amiti and Weinstein (2013) develop a new methodology to identify bank-supply shocks using matched bank–firm credit data. We show, using the Frisch–Waugh theorem, that their methodology is equivalent to a weighted least squares regression and suggest applicability in other research areas.

1. Introduction

There is a long and well-established tradition in the empirical banking literature of disentangling credit demand and supply shocks (see e.g. Khwaja and Mian, 2008; Jiménez et al., 2012; Del Giovane et al., 2011). Amiti and Weinstein (2013), henceforth AW, develop a new methodology to separate both channels using matched bank–firm credit data. The contribution of their procedure is to account for general equilibrium constraints such that the micro and macro features of the data are mutually consistent. Recently, their methodology has gained popularity among scholars (e.g. Amador and Nagengast, 2016; Degryse et al., 2016; Flannery and Lin, 2016; Tu, 2016, etc.).

Although appealing, their estimation procedure as described in their Appendix A requires, inter alia, storing large sparse matrices in the software workspace (which is problematic if sparse coding is unavailable). Moreover, it is required to cast the data in particular matrix specifications and to program the estimator from scratch. Our contribution is to formally show that these drawbacks can be circumvented; we prove that the AW estimation results can be obtained from a more convenient weighted least squares regression (WLS) applied directly to the data. More precisely, a weighted regression of bank-firm loan growth on bank and firm fixed effects (with weights equal to the lagged loan size in economywide credit) delivers the same bank shocks as the framework derived in AW. Degryse et al. (2016) notice this equivalence in their application, but do not formally prove it. This insight is useful as most statistical software packages have built-in routines for WLS, rendering WLS more suitable for shock identification than the AW framework, without losing its appealing properties.

This increased ease of use benefits other areas in economics besides the empirical finance literature; we show that the estimator can be applied in any area whenever one wants to disentangle demand and supply shocks using bilateral data while simultaneously accounting for general equilibrium considerations.

* Correspondence to: KU Leuven, Faculty of Economics and Business Economics, Department of Economics, Naamsestraat 69, 3000 Leuven, Belgium
E-mail address: joris.tielens@kuleuven.be (J. Tielens).

http://dx.doi.org/10.1016/j.econlet.2016.11.039
0165-1765/

The remainder of the paper is structured as follows. In Section 2.1 we briefly reintroduce the intuition of the AW estimator. In Section 2.2 we prove equivalence between the AW and the WLS framework and discuss its applicability beyond the empirical finance literature. Section 3 concludes. Throughout we adhere to the notation in AW.

2. The empirical framework

2.1. The Amiti–Weinstein estimator

Let L_{fbt} denote borrowing by firm f from bank b at time t. A general class of empirical models describes firm–bank loan growth as

$$\frac{L_{fbt}-L_{fbt-1}}{L_{fbt-1}} = \alpha_{ft} + \beta_{bt} + \varepsilon_{fbt} \tag{1}$$

where α_{ft} is labelled the *firm-borrowing channel* and β_{bt} is the *bank-lending channel* (see Khwaja and Mian, 2008). One approach to identify both channels is to empirically estimate (1) in a regression framework that is saturated with firm and bank fixed effects. AW show this procedure to be inefficient because it ignores general equilibrium considerations. E.g. a firm cannot borrow more without at least one bank willing to lend more (and vice versa). In the context of (1), there are general equilibrium linkages between $\{\alpha_{ft}\}_{f=1}^{F}$ and $\{\beta_{bt}\}_{b=1}^{B}$.

AW therefore propose an estimation procedure that exploits adding-up constraints implicit in (1). To see this more clearly, multiply both sides of Eq. (1) with lagged share of lending to firm f, ϕ_{fbt-1}, and sum across all firms in order to obtain

$$D_{bt}^{B} \equiv \sum_{f=1}^{F}\Big(\frac{L_{fbt}-L_{fbt-1}}{L_{fbt-1}}\Big)\frac{L_{fbt-1}}{\sum_{f=1}^{F} L_{fbt-1}} = \beta_{bt} + \sum_{f=1}^{F}\phi_{fbt-1}\alpha_{ft} + \sum_{f=1}^{F}\phi_{fbt-1}\epsilon_{fbt}$$

where $\phi_{fbt-1} \equiv \frac{L_{fbt-1}}{\sum_{f=1}^{F} L_{fbt-1}}$. D_{bt}^{B} denotes the growth rate of lending of bank b to all its borrowing firms. In a similar vein we can multiply (1) with the lagged share of borrowing from bank b, θ_{fbt-1} and sum across all banks in order to obtain

$$D_{ft}^{F} \equiv \sum_{b=1}^{B}\Big(\frac{L_{fbt}-L_{fbt-1}}{L_{fbt-1}}\Big)\frac{L_{fbt-1}}{\sum_{b=1}^{B} L_{fbt-1}} = \alpha_{ft} + \sum_{b=1}^{B}\theta_{fbt-1}\beta_{bt} + \sum_{b=1}^{B}\theta_{fbt-1}\epsilon_{fbt}$$

where $\theta_{fbt-1} \equiv \frac{L_{fbt-1}}{\sum_{b=1}^{B} L_{fbt-1}}$. D_{ft}^{F} denotes the growth rate of borrowing of firm f from all of its banks.

If one imposes the moment conditions $\mathbb{E}[\sum_{f=1}^{F}\phi_{fbt-1}\epsilon_{fbt}] = \mathbb{E}[\sum_{b=1}^{B}\theta_{fbt-1}\epsilon_{fbt}] = 0$, then one can choose the parameters such that the following system of $F+B$ equations holds in the data

$$D_{bt}^{B} = \beta_{bt} + \sum_{f=1}^{F}\phi_{fbt-1}\alpha_{ft} \tag{2a}$$

$$D_{ft}^{F} = \alpha_{ft} + \sum_{b=1}^{B}\theta_{fbt-1}\beta_{bt}. \tag{2b}$$

The contribution of the AW framework is to obtain estimates of $\{\alpha_{ft}\}_{f=1}^{F}$ and $\{\beta_{bt}\}_{b=1}^{B}$ that account both for (i) the microstructure of the data in (1) as well as (ii) the general equilibrium conditions in (2a) and (2b) which are at the aggregate level.

2.2. Weighted least squares

We first recast the above set-up from AW in matrix notation.

Let $\boldsymbol{Y_t} \in \mathbb{R}^{F\times B}$ be the matrix of firm–bank loan growth at time t, that is $[\boldsymbol{Y_t}]_{fb} = \frac{L_{fbt}-L_{fbt-1}}{L_{fbt-1}}$ and $[\boldsymbol{Y_t}]_{fb} = 0$ if firm f does not borrow from bank b at time $t-1$, i.e. $L_{fbt-1} = 0$. Let $\boldsymbol{y_t} = Vec(\boldsymbol{Y_t'}) \in \mathbb{R}^{(F\times B)\times 1}$. Furthermore, define a dummy variable $d_{fbt} = 1$ if $L_{fbt-1} \neq 0$

$$d_{fbt} = \begin{cases} 1 & \text{if } L_{fbt-1} \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

and $\boldsymbol{d_{ft}} \in \mathbb{R}^{B\times 1}$

$$\boldsymbol{d_{ft}} = \begin{bmatrix} d_{f1t} \\ \vdots \\ d_{fBt} \end{bmatrix}.$$

Define the following matrices

$$\boldsymbol{S_{1t}^{*}} = \begin{bmatrix} \boldsymbol{d_{1t}} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{d_{2t}} & & \\ \vdots & & \ddots & \vdots \\ \boldsymbol{0} & \cdots & & \boldsymbol{d_{Ft}} \end{bmatrix}, \boldsymbol{S_{1t}} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{d_{2t}} & \boldsymbol{0} & & \vdots \\ \boldsymbol{0} & \ddots & & \\ \vdots & & & \\ \boldsymbol{0} & \cdots & & \boldsymbol{d_{Ft}} \end{bmatrix}$$

$$\boldsymbol{S_{2t}} = \begin{bmatrix} diag(\boldsymbol{d_{1t}}) \\ \vdots \\ diag(\boldsymbol{d_{Ft}}) \end{bmatrix}, \boldsymbol{S_{3t}} = \begin{bmatrix} \boldsymbol{d_{1t}} \\ \vdots \\ \boldsymbol{d_{Ft}} \end{bmatrix}, \boldsymbol{\alpha_t^{*}} = \begin{bmatrix} \alpha_{1t} \\ \vdots \\ \alpha_{Ft} \end{bmatrix},$$

$$\boldsymbol{\alpha_t} = \begin{bmatrix} \alpha_{2t} \\ \vdots \\ \alpha_{Ft} \end{bmatrix}, \boldsymbol{\beta_t} = \begin{bmatrix} \beta_{1t} \\ \vdots \\ \beta_{Bt} \end{bmatrix}$$

with dimensions $\boldsymbol{S_{1t}^{*}} \in \mathbb{R}^{(F\times B)\times F}$, $\boldsymbol{S_{1t}} \in \mathbb{R}^{(F\times B)\times(F-1)}$, $\boldsymbol{S_{2t}} \in \mathbb{R}^{(F\times B)\times B}$, $\boldsymbol{S_{3t}} \in \mathbb{R}^{(F\times B)\times 1}$, $\boldsymbol{\alpha_t^{*}} \in \mathbb{R}^{F\times 1}$, $\boldsymbol{\alpha_t} \in \mathbb{R}^{(F-1)\times 1}$, $\boldsymbol{\beta_t} \in \mathbb{R}^{B\times 1}$ and lastly $\boldsymbol{0} \in \mathbb{R}^{B\times 1}$ is a vector of zeros. $\boldsymbol{S_{1t}}$ is equal to $\boldsymbol{S_{1t}^{*}}$ with the first column removed.

The following equation recasts expression (1) from AW in matrix notation.

$$\boldsymbol{y_t} = \boldsymbol{S_{1t}^{*}}\boldsymbol{\alpha_t^{*}} + \boldsymbol{S_{2t}}\boldsymbol{\beta_t} + \boldsymbol{S_{3t}}\boldsymbol{\varepsilon_t}.$$

Note that this system is perfectly collinear. Following AW, we take α_{1t} to be the numéraire ($\alpha_{1t} = 0$), and rewrite the system as[1]

$$\boldsymbol{y_t} = \boldsymbol{S_{1t}}\boldsymbol{\alpha_t} + \boldsymbol{S_{2t}}\boldsymbol{\beta_t} + \boldsymbol{S_{3t}}\boldsymbol{\varepsilon_t}. \tag{3}$$

We now algebraically reconstruct the WLS estimators of (3), $\boldsymbol{\alpha_t}(\boldsymbol{W_{t-1}})$ and $\boldsymbol{\beta_t}(\boldsymbol{W_{t-1}})$, where $\boldsymbol{W_{t-1}}$ is a weight matrix.

First, let $\boldsymbol{W_{t-1}} \in \mathbb{R}^{(F\times B)\times(F\times B)}$ be a diagonal matrix with diagonal entries equal to $w_{ii} = \frac{L_{fbt-1}}{\sum_{f=1}^{F}\sum_{b=1}^{B} L_{fbt-1}}$ and zero otherwise. Hence, the diagonal elements in $\boldsymbol{W_{t-1}}$ capture the weight of credit of bank b to firm f at time $t-1$ in aggregate credit at time $t-1$.

Next, recall that a WLS estimation of (3) with weights $\boldsymbol{W_{t-1}}$ boils down to an ordinary least squares estimation of $\boldsymbol{W_{t-1}^{\frac{1}{2}}}\boldsymbol{y_t} = \boldsymbol{W_{t-1}^{\frac{1}{2}}}\boldsymbol{S_{1t}}\boldsymbol{\alpha_t} + \boldsymbol{W_{t-1}^{\frac{1}{2}}}\boldsymbol{S_{2t}}\boldsymbol{\beta_t} + \boldsymbol{W_{t-1}^{\frac{1}{2}}}\boldsymbol{S_{3t}}\boldsymbol{\varepsilon_t}$. We use this insight, in addition to the Frisch and Waugh (1933) theorem which allows us to provide separate expressions for the coefficients estimates of $\boldsymbol{\alpha_t}(\boldsymbol{W_{t-1}})$ and $\boldsymbol{\beta_t}(\boldsymbol{W_{t-1}})$.

[1] This is the fixed effects specification estimated by OLS in Section 4.1 in AW.