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Respect for experts vs. respect for unanimity: The liberal paradox in probabilistic opinion pooling

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HIGHLIGHTS

- A formal framework for discussing expert rights in the context of probabilistic opinions is developed.
- An analogue of Amartya Sen's paradox of a Paretian liberal is formulated and proved.
- Escape strategies for avoiding the paradox are discussed.

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ABSTRACT

Amartya Sen (1970) has shown that three natural desiderata for social choice rules are inconsistent: universal domain, respect for unanimity, and respect for some minimal rights—which can be interpreted as either "expert rights" (an expert's right to have her competence respected) or liberal rights. Dietrich and List (2008) have generalised this result to the setting of binary judgement aggregation. This paper proves that the paradox of a Paretian liberal holds even in the framework of probabilistic opinion pooling and discusses options to circumvent this impossibility result: (i) restricting the aggregator domain to profiles with no potential for conflicting rights; (ii) avoiding agendas where all issues are pairwise entangled (interdependent).

1. Introduction

Beliefs within a group are often diverse. A social planner may wish to accept the privileged epistemic status of certain individuals in some areas and also accept unanimous judgements. For example, suppose Dr A is a specialist for the treatment of some medical condition α, while Dr B specialises on therapies for condition β. Based on substantial expertise, Dr A asserts that a one-year treatment with a certain drug D cures patients with the α diagnosis with probability $75 \pm 3\%$, while Dr B testifies that the chances for D to cure β patients within twelve months are merely $63 \pm 4\%$. However, both experts A and B state, citing theoretical pharmacological reasons, that D should be more effective in the treatment of β than of α. If we accept this unanimous theoretical testimony of both physicians, we cannot also accept both of their empirical expert testimonies without running into contradictions.

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More generally speaking, consider a panel of experts each making probabilistic judgements about a given set of propositions, according to their expertise. All of the experts have their specific areas of competence where they have superior expertise to their colleagues. Is it possible to aggregate their probabilistic judgements in a way that preserves unanimous agreement and follows each expert in her respective area of competence? This paper shows that this is not generally possible—even if one allows for full rationality of the experts (in the sense that they each have a fully specified probability measure on the algebra of propositions under consideration). We shall identify a set of necessary and sufficient conditions, showing that the predicament in the above example is not exceptional. For the case of binary preferential judgements this phenomenon is well-known and commonly referred to as Sen's (1970) *liberal paradox*. An analogous result for general binary judgements has been proved by Dietrich and List (2008).

The last decade has seen the emergence of a new strand of literature on general, logical aggregation problems, commonly referred to as *judgement aggregation* (surveyed, e.g., by List and Puppe, 2009 and Mongin, 2012). In distinction from Arrow's (1963) classical theory of preference aggregation, this new current of research investigates the aggregation of more general data, such as sets of propositions in some formal logic.

One of the more recent developments is a unified approach to judgement aggregation and probabilistic opinion pooling (in McConway's (1981) sense), first proposed by Dietrich and List (2010a) and subsequently investigated by other authors.[1] This kind of unification motivates a new research direction in the theory of collective decision making: Which (im)possibility results from (preference and) judgement aggregation theory possess an analogue in the framework of probabilistic pooling? The present paper contributes to such a new line of research.

The structure of the paper is as follows. We first propose a simplified framework for probabilistic opinion pooling which can be seen as a natural extension of judgement aggregation—with continuous rather than binary truth values. In this simplified framework for probabilistic opinion pooling, we formulate and prove an analogue of Sen's liberal paradox.

2. Formal framework

Let $\mathcal{A}$ be a Boolean algebra, called *agenda*, with maximal element $\top$ and minimal element $\bot$. Elements of $\mathcal{A}$ will be called *propositions*.

Let $\Delta(\mathcal{A})$ be the set of all finitely-additive probability measures on $\mathcal{A}$, called *complete probabilistic judgements*. Let $\mathcal{C}$ be a subset of $\Delta(\mathcal{A})$, called the set of *admissible probabilistic judgements*.[2]

A subset $\mathcal{C}' \subseteq \mathcal{C}$ is called *cylindrical* or *generated by proposition-wise constraints* if and only if there is a subset $\mathcal{B} \subseteq \mathcal{A}$ and a $\mathcal{B}$-indexed family $\langle \Gamma_A \rangle_{A\in\mathcal{B}}$ of subsets of $[0, 1]$, such that $\mathcal{C}'$ is the set of all $P \in \mathcal{C}$ satisfying $P(A) \in \Gamma_A$ for all $A \in \mathcal{B}$, formally: $\mathcal{C}' = \{P \in \mathcal{C} : \forall A \in \mathcal{B} \quad P(A) \in \Gamma_A\}$; in this case, $(\Gamma_A)_{A\in\mathcal{B}}$ is called the *constraint system generating* $\mathcal{C}'$ within $\mathcal{C}$.[3]

Let N be a finite or infinite set, the *electorate*, whose elements are called *individuals*.

The elements of $\mathcal{C}^N$ are called *profiles*, and the shorthand for typical elements of $\mathcal{C}^N$ will be $\underline{P}$ (in place of $(P_i)_{i\in N}$).

An *aggregator* is a map with domain $\subseteq \mathcal{C}^N$ and range $\subseteq \mathcal{C}$. An aggregator F has $\mathcal{C}$-*universal domain* if and only if $F : \mathcal{C}^N \to \mathcal{C}$. An aggregator F is said to *preserve unanimity* if and only if for all $A \in \mathcal{A}$, one has $F(\underline{P})(A) = \alpha$ whenever $P_i(A) = \alpha$ for all $i \in N$.

An individual i is *decisive* under an aggregator F for a proposition $A \in \mathcal{A}$ if and only if $F(\underline{P})(A) = P_i(A)$ for all $\underline{P}$ in the domain of F.

An aggregator F is said to *respect expert competence* if and only if there are at least two individuals that are decisive under F on at least one proposition each.

Two propositions $A, B \in \mathcal{A}$ are called $\mathcal{C}$-*entangled* if and only if for some $\alpha, \beta \in [0, 1]$ and some cylindrical $\mathcal{C}' \subseteq \mathcal{C}$ there exist $P, P' \in \mathcal{C}'$ with $P(A) = \alpha$ and $P'(B) = \beta$, but there is no $P^* \in \mathcal{C}'$ satisfying both $P^*(A) = \alpha$ and $P^*(B) = \beta$. Otherwise, A, B are called $\mathcal{C}$-*disentangled*. The agenda $\mathcal{A}$ is said to be $\mathcal{C}$-*entangled* if and only if all pairs of propositions $A, B \in \mathcal{A} \setminus \{\bot, \top\}$ are $\mathcal{C}$-entangled.[4]

The notion of $\mathcal{C}$-entanglement is novel and thus merits some discussion. Two propositions A, B are entangled if one can find a set of proposition-wise generated constraints on probability measures, encoded by $\mathcal{C}' \subseteq \mathcal{C}$, and a pair of probability assignments to A, B which are independently but not jointly compatible with $\mathcal{C}'$. Conversely, A, B are $\mathcal{C}$-disentangled if and only if for all proposition-wise constrained sub-families $\mathcal{C}'$ of $\mathcal{C}$ and all $\alpha, \beta \in [0, 1]$ one has the following implication: Whenever the probability assignments $P(A) = \alpha$ and $P(B) = \beta$ are each consistent with the constraints $\mathcal{C}'$, then they are also jointly consistent with $\mathcal{C}'$ (i.e. there exists a single $P^* \in \mathcal{C}'$ with both $P^*(A) = \alpha$ and $P^*(B) = \beta$). Calling such a pair of propositions A, B disentangled is justified by the observation that assigning a given probability to A does not determine the probability assignment to B (at least not beyond any limitations imposed by $\mathcal{C}$).

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[1] Two years after circulating the first drafts of this paper, the author was informed that there is even an analogue of Sen's liberal paradox in Dietrich and List's unified approach to judgement aggregation and probabilistic opinion pooling. This result is still unpublished, but was discussed in Dietrich and List's *Social Choice and Welfare Prize Lecture* (Dietrich and List, 2010b), as this author recently learned from an anonymous referee.

[2] $\mathcal{C}$ can be viewed as encoding a common background theory shared by the electorate (which may include the requirement of σ-additivity for the individual probability measures).

In our motivating example, the common background theory would be the theoretical pharmacological statement D should be more effective in the treatment of β than of α. $\mathcal{C}$ would consist of all finitely additive probability measures on some fixed set of possible worlds that are consistent with this theoretical statement.

An extreme case would be $\mathcal{C} = \Delta(\mathcal{A})$, which means that every complete probabilistic judgement is treated as admissible, which means that there is no background theory shared by the electorate.

The opposite extreme case is that the commonly shared background theory is so rigid that it fully determines the probabilistic judgements, whence the set of admissible probabilistic judgements is a singleton, $\mathcal{C} = \{P_c\}$, say. (In other words, everyone in the electorate agrees that the only admissible probabilistic judgement is P_c.)

The interesting cases are, of course, those where $\mathcal{C}$ is neither a singleton nor the entire set $\mathcal{C} = \Delta(\mathcal{A})$. To give a simple example, it could be common knowledge among the electorate that a certain event E (e.g., that a given fair coin lands heads) has probability 1/2. If this is all that the electorate is agreed upon, the set of admissible probabilistic judgements would be $\mathcal{C} = \{P \in \Delta(\mathcal{A}) : P(E) = 1/2\}$.

[3] In other words, $\mathcal{C}'$ is generated by proposition-wise constraints if and only if it coincides with the set of all those probability measures that assign only particular choices of probabilities to a given collection of sets.

For instance, fix $A, A' \in \mathcal{A}$. Then the set of all P such that $P(A) = 0$ and $\frac{1}{2} \le P(A') \le 1$ is generated by proposition-wise constraints, viz. constraints on the probabilities of propositions A and A'. (To see this, let $\mathcal{B} = \{A, A'\}$ and $\Gamma_A = \{0\}$ as well as $\Gamma_{A'} = [0.5, 1]$.)

For another example, let $\mathcal{B} \subseteq \mathcal{A}$ and suppose Γ_A is an interval for all $A \in \mathcal{B}$. Consider the set $\mathcal{C}' = \{P \in \Delta(\mathcal{A}) : P \quad \sigma\text{-additive} \;\&\; \forall A \in \mathcal{B} \quad P(A) \in \Gamma_A\}$, i.e. the set of ($\sigma$-additive) probability measures which is generated by the constraint system $(\Gamma_A)_{A\in B}$. Then $\mathcal{C}'$ is an interval-valued probability measure.

[4] For two propositions A, B to be $\mathcal{C}$-entangled roughly means that they are 'conditionally dependent' in the sense that assigning a probability to A already restricts the possibilities of assigning a probability to B. Thus, the notion of entanglement of an agenda corresponds to the condition of agenda connectedness known from judgement aggregation theory, which also stipulates that any two propositions in the agenda must be conditionally dependent. (In the judgement-aggregation literature, conditional dependence is usually defined in terms of the existence of certain minimally inconsistent agenda subsets that enjoy additional properties.)