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Weak differential monotonicity, flat tax, and basic income[☆]

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HIGHLIGHTS

- We reconsider Casajus' (Econom Lett, 2016) foundation of flat tax and basic income.
- That is, redistribution via fixed-rate taxation and equal sharing of the tax revenue.
- We relax Casajus' differential monotonicity into weak differential monotonicity.
- As Casajus, we also use efficiency and non-negativity, but add an average property.
- These properties characterize the afore-mentioned class of redistribution rules.

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ABSTRACT

We suggest a weak version of differential monotonicity for redistribution rules: whenever the differential of two persons' income weakly increases, then their post-redistribution rewards essentially change in the same direction. Together with efficiency, non-negativity, and the average property, weak differential monotonicity characterizes redistribution via taxation at a fixed rate and equal distribution of the total tax revenue, i.e., a flat tax and a basic income.

1. Introduction

In modern societies, their members' income[1] is redistributed via various channels. A simple model to study redistribution of income in an n-member society is the following: Its members are numbered from 1 to n; $\mathbb{N}_n := \{1, \ldots, n\}$. Their incomes are given by a vector $x \in \mathbb{R}^n$. Redistribution is modelled by mappings (redistribution rules) $f : \mathbb{R}^n \to \mathbb{R}^n$.[2] For $x \in \mathbb{R}^n$ and $i \in \mathbb{N}_n$, $f_i(x)$ denotes member i's reward after redistribution.

Monotonicity principles or properties have a long tradition and are ubiquitous in economics and game theory (see e.g. Sprumont, 2008). Casajus (2016) suggests a differential version of monotonicity for redistribution rules: whenever the differential of two persons' income weakly increases, then the differential of their post-redistribution rewards also weakly increases. For societies

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1 In this paper, the term income embraces both gains and losses.

2 Throughout, we disregard the trivial case $n = 1$.

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comprising more than two members, differential monotonicity together with efficiency and non-negativity characterizes redistribution via taxation at a fixed rate and equal distribution of the total tax revenue, i.e., flat tax combined with a basic income but which depends on the total income (his Theorem 4 and Remark 5). This does not hold true for two-member societies (his Counterexample).

In this paper, we explore a substantial relaxation of differential monotonicity called *weak differential monotonicity* and explore its implications. As its stronger cousin, this property deals with situations where the differential of two members' income weakly increases, put differently, one player's change in income is weakly greater than that of a second one. In such situations, weak differential monotonicity requires the following. First, if the second player's post-redistribution reward does not change, then the former player's post-redistribution reward weakly increases. Second, if the second player's post-redistribution reward increases, then the former player's post-redistribution reward also increases. One easily checks that differential monotonicity implies weak differential monotonicity.

First, we show that one cannot replace differential monotonicity with weak differential monotonicity in Casajus (2016, Theorem 4). Second, cum grano salis, we show how the result of Casajus (2016, Theorem 4) can be restored after replacing differential monotonicity with weak differential monotonicity. To this end, we suggest the average property. This property requires that the relation of the income of a member of the society to the average income is preserved by redistribution, i.e., a member with weakly above (below) average income obtains a weakly above (below) average post-redistribution reward. As our main result, we show that for societies with at least four members the average property together with weak differential monotonicity, efficiency, and non-negativity implies redistribution via a flat tax and a (uniform) basic income paid from the total tax revenue (Theorem 2).[3]

This paper is organized as follows. The second section presents Casajus' (2016) benchmark result on differential monotonicity. In the third section, we introduce and discuss weak differential monotonicity. The fourth section contains our main result on weak differential monotonicity, the flat tax, and a basic income. An Appendix contains the proof of our main result.

2. Differentially monotonic redistribution of income

In this section, we provide the benchmark result on differentially monotonic redistribution.

Efficiency, E. For all $x \in \mathbb{R}^n$, we have $\sum_{\ell=1}^{n} f_\ell(x) = \sum_{\ell=1}^{n} x_\ell$.

The very idea of *re*-distribution suggests that the total rewards after redistribution should not be greater than total income before. In addition, efficiency requires that redistribution has no cost.

Non-negativity, NN. For all $x \in \mathbb{R}^n_+$ and $i \in \mathbb{N}_n$, we have $f_i(x) \geq 0$.[4]

Non-negativity is a very natural requirement. For non-negative income, no member of the society necessarily must end up with a negative post-redistribution reward.

Differential monotonicity, DM. For all $x, y \in \mathbb{R}^n$ and $i, j \in \mathbb{N}_n, i \neq j$ such that $x_i - x_j \geq y_i - y_j$, we have $f_i(x) - f_j(x) \geq f_i(y) - f_j(y)$.

This property demands non-decreasing income differentials of two members to translate into non-decreasing differentials of their post-redistribution rewards.

For societies comprising more than two members, Casajus (2016, Theorem 4 and Remark 5) shows that the afore-mentioned properties characterize redistribution via taxation at a fixed rate and equal distribution of the total tax revenue, i.e., a flat tax combined with a basic income but which depends on the total income.

Theorem 1 (*Casajus, 2016*)**.** *Let $n > 2$. A redistribution rule $f : \mathbb{R}^n \to \mathbb{R}^n$ satisfies efficiency (**E**), non-negativity (**NN**), and differential monotonicity (**DM**) if and only if there exists some $\tau \in [0, 1]$ such that*

$$f_i(x) = (1-\tau) \cdot x_i + \frac{\tau}{n} \cdot \sum_{\ell=1}^{n} x_\ell \qquad \text{for all } x \in \mathbb{R}^n \text{ and } i \in \mathbb{N}_n. \quad (1)$$

3. Weak differential monotonicity

In this section, we suggest and explore a relaxation of differential monotonicity. The latter can be rephrased as follows.

Differential monotonicity, DM. For all $x, y \in \mathbb{R}^n$ and $i, j \in \mathbb{N}_n, i \neq j$ such that $x_i - y_i \geq x_j - y_j$, we have $f_i(x) - f_i(y) \geq f_j(x) - f_j(y)$.

Whenever one player's increase in income is weakly greater than another one's, the former player's increase in post-redistribution reward is weakly greater than latter one's. From this formulation it is immediate that differential monotonicity implies the following considerably weaker property.

Weak differential monotonicity, DM$^-$. For all $x, y \in \mathbb{R}^n$ and $i, j \in \mathbb{N}_n, i \neq j$ such that $x_i - y_i \geq x_j - y_j$, we have that

(i) $f_j(x) = f_j(y)$ implies $f_i(x) \geq f_i(y)$ and that

(ii) $f_j(x) > f_j(y)$ implies $f_i(x) > f_i(y)$.

In contrast to differential monotonicity, which has implications for post-redistribution reward differentials, its weaker cousin only requires the direction of post-redistribution reward changes to reflect pre-redistribution income differences.

First, we show that one cannot replace differential monotonicity with weak differential monotonicity in Theorem 1. Set $\Delta^n_{++} := \left\{\sigma \in \mathbb{R}^n \mid \sigma_i > 0 \text{ for all } i \in \mathbb{N}_n \text{ and } \sum_{\ell=1}^{n} \sigma_\ell = 1\right\}$. For $\sigma \in \Delta^n_{++}$, let the redistribution rule $f^\sigma : \mathbb{R}^n \to \mathbb{R}^n$ be given by

$$f_i^\sigma(x) = \sigma_i \cdot \sum_{\ell=1}^{n} x_\ell \qquad \text{for all } x \in \mathbb{R}^n \text{ and } i \in \mathbb{N}_n. \quad (2)$$

One easily checks that the redistribution rules f^σ satisfies efficiency, non-negativity, and weak differential monotonicity. However, these redistribution rules are not as in Theorem 1 as long as σ does not assign the same weight to any of the members.

4. Weak differential monotonicity, flat tax, and basic income

In this section, cum grano salis, we show how the result of Theorem 1 can be restored after replacing differential monotonicity with weak differential monotonicity. Consider the following property.

Average, AV. For all $x \in \mathbb{R}^n$ and $i \in \mathbb{N}_n$, we have that

(i) $x_i \geq \frac{1}{n} \cdot \sum_{\ell=1}^{n} x_\ell$ implies $f_i(x) \geq \frac{1}{n} \cdot \sum_{\ell=1}^{n} f_\ell(x)$ and

(ii) $x_i \leq \frac{1}{n} \cdot \sum_{\ell=1}^{n} x_\ell$ implies $f_i(x) \leq \frac{1}{n} \cdot \sum_{\ell=1}^{n} f_\ell(x)$.

This property requires that the relation of the income of a member of the society to the average income is (weakly) preserved by redistribution. That is, a member with a weakly above (below) average income obtains a weakly above (below) average post-redistribution reward. Cum grano salis, this property can be justified as a means to incentivize members to earn higher incomes.

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[3] The flat tax (or proportional tax) has been advocated in 1845 by McCulloch (1975) and later on by notable others as Mill (1848), Hayek (1960), and Friedman (1962), more recently by Hall and Rabushka (1985) and Hall (1996). Vanderborght and Van Parijs (2005) provide a survey on the idea of an unconditional basic income. The idea of a flat tax combined with an unconditional basic income that depends on the total productivity of the society was suggested by Milner (1920), for example.

[4] We set $\mathbb{R}_+ := [0, +\infty)$.