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The profit function system with output- and input-specific technical efficiency

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ABSTRACT

In a recent paper Kumbhakar and Lai (2016) proposed an output-oriented non-radial measure of technical inefficiency derived from the revenue function. They proposed a closed skew-normal distribution for maximum likelihood estimation but they did not apply the model to data and their technique depends on multiple evaluations of multivariate normal integrals for each observation which can be very costly. In this paper we extend their approach to the profit function and we propose both input- and output-oriented non-radial measures of technical inefficiencies. Although the extension to the translog profit function is trivial many observations, in practice, may contain negative profits. For this reason we provide a nontrivial extension to the Symmetric Generalized McFadden (SGM) profit function. We propose and apply (to a large sample of US banks) Bayesian analysis of the SGM model (augmented with latent technical inefficiencies resulting in a highly nonlinear mixed effects model) using the integrated nested Laplace approximation.

1. Introduction

In this paper we build on Kumbhakar and Lai (2016) who developed an output-specific (vector) efficiency measure starting from the revenue function and using the envelope theorem to obtain the output shares. They have used the translog revenue function and, under the assumption that latent output-specific inefficiencies follow a multivariate half-normal distribution they obtained the likelihood function of the system of output shares (without the translog revenue function) using a closed skew-normal distribution. Kumbhakar and Lai (2016) did not take the model to the data. A difficulty that arises in their likelihood is that it requires multiple evaluations of multivariate normal integrals for each observation which can be very costly.

In this study we extend Kumbhakar and Lai (2016) to the profit function case in order to derive both output-specific and input-specific inefficiency measures. As the authors mention: "Although in the present model we consider only output slacks, the formulation can be extended to accommodate both input and output slacks in a profit maximizing model". This is true but certain nontrivial problems arise. In order to get rid of the awkward normalizing constants of the closed skew-normal distribution we propose a multivariate lognormal distribution for the latent input and output inefficiencies. Second, the extension of revenue to profit functions requires that all observations have positive profits which is rarely the case. Therefore, we adopt a Symmetric Generalized McFadden (SGM) profit function. We propose and apply (to a large sample of US banks) Bayesian analysis of the SGM model (augmented with latent technical inefficiencies resulting in a highly nonlinear mixed effects model) using the integrated nested Laplace approximation. To our knowledge this is the first study that analyzes the SGM profit function enforcing all regularity restrictions globally without calibrating certain parameters.

2. Model

We build on Kumbhakar and Lai (2016) to construct a profit system with both output- and input-oriented inefficiency. The vector of netputs is $z \in \Re^N$, assuming outputs are positive and inputs are negative. For simplicity $z_1, \ldots, z_M > 0$ are outputs and $z_{M+1}, \ldots, z_N < 0$ are inputs. Prices are $p \in \Re^N_+$. The objective of the firm is profit maximization:

$$\max_{z \in \Re^N} : \; p^\top z, \;\; \text{s.t.}\; F(z^*) = 1, \tag{1}$$

where $z^* = \theta \odot z, \theta \in \Re^N$ with $\theta_1, \ldots, \theta_M \geq 1$ and $\theta_{M+1}, \ldots, \theta_N \leq 1$. The problem is equivalent to:

$$\Pi(p_*) = \max_{z \in \Re^N} : \; p_*^\top z, \;\; \text{s.t.}\; F(z) = 1, \tag{2}$$

where $p_* = [p_n/\theta_n, n = 1, \ldots, N]$ in view of equations (2) in Kumbhakar and Lai (2016). Using the envelope theorem we have

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the netput demands in the form: $z = \frac{\partial \Pi(p_*)}{\partial p_*}$. Alternatively we have the shares: $\frac{\partial \log \Pi(p_*)}{\partial \log p_n^*} = \frac{p_n^* z_n}{\Pi(p_*)}$, $n = 1, \dots, N$. Next we assume a translog profit function:

$$\log \Pi(p_*) = \beta_0 + \sum_{n=1}^{N} \beta_n \log p_n^* + \tfrac{1}{2} \sum_{n=1}^{N} \sum_{m=1}^{N} \beta_{nm} \log p_n^* \log p_m^*. \quad (3)$$

From the envelope theorem we obtain:

$$\frac{\partial \log \Pi(p_*)}{\partial \log p_n^*} = \beta_n + \sum_{m=1}^{n} \beta_{nm} \left(\log p_m - \log \theta_m\right), n = 1, \dots, N. \quad (4)$$

Defining $\xi_n = -\log \theta_n$, $n = 1, \dots, N$ we have the following system of equations:

$$\begin{aligned} \log \Pi(p_*) &= \beta_0 + \sum_{n=1}^{N} \beta_n \log p_n + \tfrac{1}{2} \sum_{n=1}^{N} \sum_{m=1}^{N} \beta_{nm} \log p_n \log p_m \\ &\quad + A(p, \xi) + v_0, \\ S_n &= \frac{p_n z_n}{\Pi} = \beta_n + \sum_{m=1}^{n} \beta_{nm} \log p_m + \sum_{m=1}^{N} \beta_{nm} \xi_m + v_n, \\ &\quad n = 1, \dots, N-1, \end{aligned} \quad (5)$$

where

$$\begin{aligned} A(p, \xi) &= \sum_{n=1}^{N} \beta_n \xi_n + \sum_{n=1}^{N} \sum_{m=1}^{N} \beta_{nm} \log p_m \xi_n \\ &\quad + \tfrac{1}{2} \sum_{n=1}^{N} \sum_{m=1}^{N} \beta_{nm} \xi_n \xi_m, \end{aligned} \quad (6)$$

and $\mathbf{v} = [v_0, v_1, \dots, v_{N-1}]$ are error terms satisfying $\mathbf{v} \sim \mathcal{N}_N(O, \Sigma)$. For the ξ_ns we have a multivariate half-normal distribution independently of $\mathbf{v}$ and prices:

$$[\xi_1, \dots, \xi_M, -\xi_{M+1}, \dots, -\xi_N] \sim \mathcal{N}_N^+(O, \Omega). \quad (7)$$

As $\sum_{n=1}^{N} S_n = 1$ we can omit the last share equation. To impose homogeneity of degree one in prices we can employ the usual parametric restrictions or redefine $p_n := p_n/p_1$ in which case the system in (5) takes the form:

$$\begin{aligned} \log \Pi(p_*) &= \beta_0 + \sum_{n=2}^{N} \beta_n \log p_n + \tfrac{1}{2} \sum_{n=2}^{N} \sum_{m=2}^{N} \beta_{nm} \log p_n \log p_m \\ &\quad + A(p, \xi) + v_0, \\ S_n &= \frac{p_n z_n}{\Pi} = \beta_n + \sum_{m=2}^{n} \beta_{nm} \log p_m + \sum_{m=1}^{N} \beta_{nm} \xi_m + v_n, \\ &\quad n = 1, \dots, N-1, \end{aligned} \quad (8)$$

where

$$\begin{aligned} A(p, \xi) &= \sum_{n=1}^{N} \beta_n \xi_n + \sum_{n=1}^{N} \sum_{m=2}^{N} \beta_{nm} \log p_m \xi_n \\ &\quad + \tfrac{1}{2} \sum_{n=1}^{N} \sum_{m=1}^{N} \beta_{nm} \xi_n \xi_m. \end{aligned} \quad (9)$$

Additionally, $\xi_1, \dots, \xi_M \leq 0$ and $\xi_{M+1}, \dots, \xi_N \geq 0$. Kumbhakar and Lai (2016) essentially consider *only* the system of the last N equations in (5) ignoring their revenue function because it is not linear in ξ. Ignoring, however, this information may be critical as the revenue or profit function provides significant identifying information.[1] Then they formulate the likelihood function from the system of share equations using properties of the closed skew-normal distribution. This involves evaluating multivariate normal integrals in $\Re^{N-1}$ for each observation which can be cumbersome and computationally non-trivial. The *entire system* in (5) can be estimated using Markov Chain Monte Carlo and especially efficient techniques developed in Kumbhakar and Tsionas (2005).

3. The Symmetric Generalized McFadden form and posterior analysis

If all profits are strictly positive we can proceed with the system in (5). In empirical applications, more often than not some observations have negative profit we have to proceed with a different functional form.[2] The Symmetric Generalized McFadden form (SGM) has been introduced by Diewert and Wales (1987) in the context of cost functions. As a profit function, the SGM takes the following form:

$$\Pi(p^o) = \sum_{n=1}^{N} \beta_n p_n^o + \tfrac{1}{2} \frac{\sum_{n=1}^{N} \sum_{m=1}^{N} \beta_{nm} p_n^o p_m^o}{\sum_{n=1}^{N} \alpha_n p_n^o}, \quad (10)$$

where $p_n^o = p_n + \theta_n$, $\forall n = 1, \dots, N$ where $\theta_1, \dots, \theta_M \geq 0$ and $\theta_{M+1}, \dots, \theta_N \leq 0$. The SGM profit function is linear homogeneous in prices. Convexity can be imposed by restricting the $[\beta_{nm}]$ matrix to positive semidefinite (e.g. by using the Cholesky decomposition) and holds globally. From the envelope theorem we have the netput demands and, after introducing error terms we have the following system:

$$\begin{aligned} \Pi(p^o) &= \sum_{n=1}^{N} \beta_n p_n^o + \tfrac{1}{2} \frac{\sum_{n=1}^{N} \sum_{m=1}^{N} \beta_{nm} p_n^o p_m^o}{\sum_{n=1}^{N} \alpha_n p_n^o}, + v_0, \\ z_n &= \beta_n + \frac{\sum_{m=1}^{N} \beta_{nm} p_m^o}{S} - \frac{1}{2} \frac{\alpha_n \sum_{n=1}^{N} \sum_{m=1}^{N} \beta_{nm} p_n^o p_m^o}{S^2} \\ &\quad + v_n, n = 1, \dots, N, \end{aligned} \quad (11)$$

where $S = \sum_{m=1}^{N} \alpha_m p_m^o > 0$, $\alpha_n \geq 0$, $\forall n = 1, \dots, N$ and $\sum_{m=1}^{N} \beta_{nm} p_m^o = 0$, $\forall n = 1, \dots, N$.

Again, we assume $\mathbf{v} = [v_0, v_1, \dots, v_N]$ are error terms satisfying $\mathbf{v} \sim \mathcal{N}_{N+1}(O, \Sigma)$ and

$$\log[\theta_1, \dots, \theta_M, -\theta_{M+1}, \dots, -\theta_N] \sim \mathcal{N}_N(\mu, \Omega), \quad (12)$$

where[3] $\mu \in \Re^{N+1}$. In "expanded" form the system is the following:

$$\begin{aligned} \Pi &= \sum_{n=1}^{N} \beta_n \left(p_n + \theta_n\right) \\ &\quad + \tfrac{1}{2} \frac{\sum_{n=1}^{N} \sum_{m=1}^{N} \beta_{nm} \left(p_n + \theta_n\right) \left(p_m + \theta_m\right)}{\sum_{n=1}^{N} \alpha_n \left(p_n + \theta_n\right)}, + v_0, \\ z_n &= \frac{\partial \Pi}{\partial p_n^o} = \beta_n + \frac{\sum_{m=1}^{N} \beta_{nm} \left(p_m + \theta_m\right)}{\sum_{m=1}^{N} \alpha_m \left(p_m + \theta_m\right)} \\ &\quad - \frac{1}{2} \frac{\alpha_n \sum_{n=1}^{N} \sum_{m=1}^{N} \beta_{nm} \left(p_n + \theta_n\right) \left(p_m + \theta_m\right)}{\left\{\sum_{m=1}^{N} \alpha_m \left(p_m + \theta_m\right)\right\}^2} + v_n, \\ &\quad n = 1, \dots, N. \end{aligned} \quad (13)$$

[1] In principle identification of the one-sided component is not a problem, unless the distribution of the overall error term turns out to be nearly symmetric. In applications this may often be the case. Therefore, the inclusion of the profit (or revenue) function may become essential as it provided information, in nonlinear form, about the one-sided components.

[2] Some authors add a constant to profits so that all of them become positive. We do not follow this arbitrary practice here.

[3] The assumption of multivariate log-normality besides being quite flexible it avoids the presence of awkward integrating constants like the multivariate normal c.d.f which would, otherwise, pose certain obstacles to both maximum likelihood as well as posterior analysis using Monte Carlo techniques.