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A critical note on Salience Theory of choice under risk

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HIGHLIGHTS

- The paper points out serious flaws in the Salience Theory model.
- The lottery certainty equivalent is undefined for some ranges of probabilities.
- Monotonicity is violated.
- The origin of the model peculiarity lies in switching between different evaluation expressions.
- The number of expressions and switching values grows rapidly with the number of states considered.

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ABSTRACT

Salience Theory Bordalo et al. (2012a) is a context-dependent theory of choice under risk, where objective probabilities are replaced by decision weights distorted in favor of salient payoffs. The detailed analysis presented in this paper points out serious flaws in this model, the most serious of which is that the lottery certainty equivalent is undefined for some ranges of probabilities. Moreover, the model violates monotonicity. The origin of the peculiar features of the model lies in switching between different evaluation expressions that depend on salience conditions or the number of prospect payoffs. The number of evaluation expressions and switching values grows rapidly with the number of states considered.

1. Introduction

The Salience Theory of choice under risk (Bordalo, Gennaioli and Shleifer—BGS, 2012a) assumes that objective probabilities are replaced by decision weights distorted in favor of salient payoffs. The authors claim that their model provides a novel and unified account of many empirical phenomena, including risk-seeking behavior, the Allais paradox, and preference reversals. The theory has been applied to e.g. asset pricing, consumer choice, and judicial decisions, and the results have been published in major economic journals (BGS, 2012b, 2013a, 2013b, 2015). This paper, however, points out a serious flaw in this model, viz. the lottery certainty equivalent (*ce*) is undefined for some ranges of probabilities (the certainty equivalent is the sure sum of money that the decision maker regards as equal to the prospect; determining *ce* was not analyzed by BGS). This peculiar feature of the model is demonstrated for two-outcome (Section 3) and three-outcome (Section 4) lotteries. Without further assumptions, then, the model cannot be used in applications that require the lottery *ce*. Moreover, the model violates monotonicity. This is best illustrated by the shape of the indifference curves in the Marschak–Machina triangle (Section 4). All things considered, despite its appealing psychological foundation, it is difficult to accept the model from either a normative or experimental viewpoint (Section 5).

2. The model

This section presents a brief summary of Salience Theory. For a more detailed exposition, please refer to BGS (2012a). According to the theory, a choice problem is described by: (1) a set of states of the world S, where each state $s \in S$ occurs with an objective and known probability[1] p_s, such that $\sum_{s \in S} p_s = 1$; and (2) a choice set $[L_1, L_2]$, where the L_1 are risky prospects that yield monetary payoffs x_s^i in each state s. The decision maker

[1] Contrary to the original BGS (2012a) paper, objective probabilities are denoted as p, rather than π.

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departs from the Expected Utility model by overweighting the lottery's most salient states in S. The salience of state s for lottery L_i and L_j ($i \neq j$) is defined[2] as a continuous and bounded function $\sigma(x_s^i, x_s^j)$, that satisfies ordering, diminishing sensitivity, and reflection. "According to the ordering property, the salience of a state for L_i increases in the distance between its payoff x_s^i and the payoff x_s^j of the alternative lottery" (BGS, page 1250). In addition, symmetry, i.e. $\sigma\left(x_s^i, x_s^j\right) = \sigma\left(x_s^j, x_s^i\right)$ "is a natural property in the case of two lotteries". BGS consider the following example salience function, where parameter $\theta > 0$:

$$\sigma\left(x_s^i, x_s^j\right) = \frac{|x_s^i - x_s^j|}{\left|x_s^i\right| + \left|x_s^j\right| + \theta}. \quad (1)$$

The salience ranking of state s is denoted $k_s : k_s \in \{1, \ldots, |S|\}$, with the lower value indicating higher salience. In a choice between two lotteries, the decision maker prefers L_1 to L_2 iff:

$$\sum_{s \in S} \delta^{k_s} p_s \left[v\left(x_s^1\right) - v\left(x_s^2\right)\right] > 0 \quad (2)$$

p_s denotes the probability of state s occurring. Parameter δ measures the extent to which salience distorts decision weights. δ is raised to the power of k_s: when $\delta = 1$, decision weights coincide with objective probabilities; when $\delta < 1$, more salient states are less discounted. Finally, v is a value (utility) function, which BSG often assume to be linear.

3. Certainty equivalent of a binary lottery

Consider a binary lottery $L_1 = \{x_1, (1-p); x_2, p\}$, where $0 \leq x_1 < x_2$. In order to determine its certainty equivalent ce, the lottery is compared with a degenerate lottery $L_{ce} = \{ce, 1\}$. The state space in the choice set $[L_1, L_{ce}]$ is $S = \{(x_1, ce), (x_2, ce)\}$. Three possible salience conditions are considered:

(1) $\sigma(x_2, ce) > \sigma(x_1, ce)$. In this case, the following expression has to be used to evaluate the preference between the lotteries:

$$V_{21} = \delta p \left[v(x_2) - v(ce)\right] + \delta^2 (1-p) \left[v(x_1) - v(ce)\right].$$

(2) $\sigma(x_1, ce) > \sigma(x_2, ce)$. The following expression has to be used:

$$V_{12} = \delta (1-p) \left[v(x_1) - v(ce)\right] + \delta^2 p \left[v(x_2) - v(ce)\right].$$

(3) $\sigma(x_2, ce) = \sigma(x_1, ce)$. Definition 2 (BGS, pp. 1251–1252) says that "all states with the same salience obtain the same ranking (and the ranking has no jumps)". According to this definition, the following expression has to be used:

$$V_{2eq1} = \delta p \left[v(x_2) - v(ce)\right] + \delta (1-p) \left[v(x_1) - v(ce)\right].$$

The certainty equivalent utility is determined by solving $V_{21} = 0$, $V_{12} = 0$, and $V_{2eq1} = 0$, as the two lotteries L_1 and L_{ce} should be indifferent:

$$v(ce) = v(x_1) + \left[v(x_2) - v(x_1)\right] \times \begin{cases} \dfrac{p}{p + \delta(1-p)} & \text{if } \sigma(x_1, ce) < \sigma(x_2, ce) \\ p & \text{if } \sigma(x_1, ce) = \sigma(x_2, ce) \\ \dfrac{\delta p}{\delta p + (1-p)} & \text{if } \sigma(x_1, ce) > \sigma(x_2, ce). \end{cases} \quad (3)$$

[2] The notation used by BGS is highly confusing because they use x_s^{-i} to denote payoffs of lottery L_j. This was therefore changed in this paper.

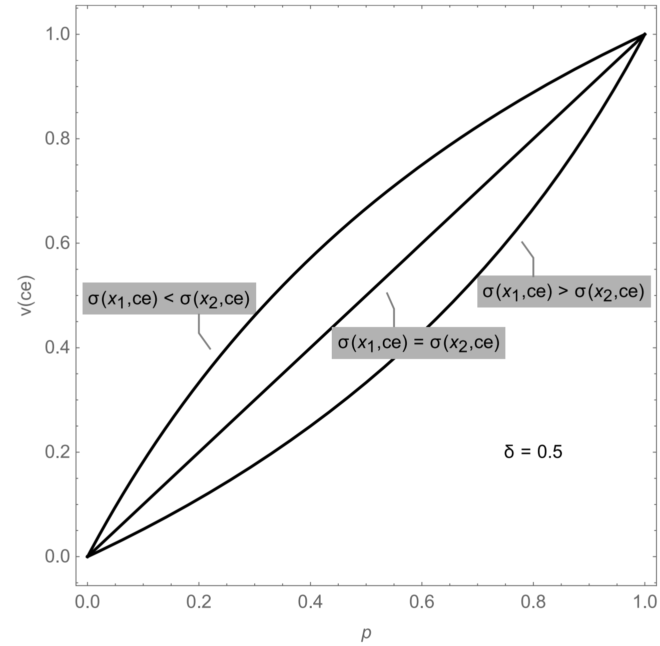

Fig. 3.1. The three shapes of the $v(ce)$ function defined in (Eq. (3)).

Without loss of generality, $v(x_1) = 0$ and $v(x_2) = 1$ can be assumed. In this case, the expression on the left of the brace assumes a value of 1, and the certainty equivalent utility $v(ce)$ is expressed by the formulas on its right. Note that $v(ce)$ belongs then to the interval [0, 1]. The shapes of the three functions defined in (Eq. (3)) are presented in Fig. 3.1. The curves are either concave, linear, or convex over the entire range of probability p. Which of these, however, applies in a given probability sub-range depends on the salience conditions.

Several observations need to be made to determine the sub-ranges. Clearly, ce belongs to the $[x_1, x_2]$ interval. Assuming ordering, increasing ce from x_1 to x_2, results in $\sigma(x_1, ce)$ increasing from $\sigma(x_1, x_1)$ to $\sigma(x_1, x_2)$, and $\sigma(x_2, ce)$ decreasing from $\sigma(x_2, x_1)$ to $\sigma(x_2, x_2)$. Thus, assuming symmetry, i.e. $\sigma(x_1, x_2) = \sigma(x_2, x_1)$, there always exists a ce_g value such that $\sigma\left(x_1, ce_g\right) = \sigma\left(x_2, ce_g\right)$ holds. If follows, that $\sigma(x_1, ce) < \sigma(x_2, ce)$ for $ce < ce_g$, and $\sigma(x_1, ce) > \sigma(x_2, ce)$ for $ce > ce_g$. ce_g is thus the ce value at which the model switches between three different evaluation expressions. Once the switching ce_g value is known, the probability sub-ranges in which the respective formulas in (Eq. (3)) apply, are determined: $[0, p_{gl})$ for the first formula; the point p_g for the second formula; and $(p_{gr}, 1]$ for the third formula, here: $p_{gl} = \frac{\delta v(ce_g)}{[1-(1-\delta)v(ce_g)]}$, $p_g = v\left(ce_g\right)$, and $p_{gr} = \frac{v(ce_g)}{[\delta+(1-\delta)v(ce_g)]}$. The resulting certainty equivalent utility $v(ce)$ as a function of probability p is presented in Fig. 3.2.

Note that the certainty equivalent utility $v(ce)$ (thus the certainty equivalent ce itself) is undefined for $p \in \left[p_{gl}, p_g\right)$ and $p \in (p_g, p_{gr}]$. *Any* assumption regarding the $v(ce)$ value in the ranges under discussion violates the one-to-one correspondence between p and $v(ce)$, as all the possible $v(ce)$ values in the range [0,1] are already assigned to probabilities from other sub-ranges. Moreover, any such assumption violates monotonicity, as better lotteries (i.e. lotteries with a greater probability of winning x_2) can clearly have smaller certainty equivalents than inferior ones. Therefore, the only sensible assumption that can make the model operational is that $v(ce)$ assumes a constant value of $v\left(ce_g\right)$ for $p \in \left[p_{gl}, p_g\right)$ and $p \in (p_g, p_{gr}]$. This also violates monotonicity, but in a weaker sense.