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Matching with contracts, substitutes and two-unit demand

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HIGHLIGHTS

- We consider the many-to-one matching with contracts model.
- We focus on choice functions that satisfy the IRC condition and two-unit demand.
- We show that weak and bilateral substitutability are equivalent.
- We obtain a new maximal domain for the existence of stable matchings with couples.

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ABSTRACT

In the context of many-to-one matching with contracts, we show that for any choice function that satisfies the irrelevance of rejected contracts condition (Aygün and Sönmez, 2013) and selects at most two contracts from any given set of contracts (two-unit demand), bilateral substitutability and weak substitutability are equivalent. As a corollary, we obtain a new maximal domain for the existence of stable matchings in the unit-capacity couples model. Finally, we show with an example that the equivalence between bilateral and weak substitutability crucially depends on the two-unit demand condition.

1. Introduction

In this note we study conditions for the existence of stable matchings in the many-to-one matching with contracts model of Hatfield and Milgrom (2005). Stability is a central concept in the matching literature. Theoretically, stable matchings are immune to rematching. Moreover, there is empirical evidence that in centralized labor markets, clearinghouses are most often successful if they produce stable matchings.[1] Unfortunately, if contracts are complements from the point of view of firms,[2] stable matchings do not always exist.

The existence of stable matchings can be guaranteed by imposing conditions on firms' choice functions. Under the assumption that each firm's choice function satisfies a mild consistency condition called irrelevance of rejected contracts (IRC) (Aygün and Sönmez, 2013), bilateral substitutability is a sufficient condition for the existence of stable matchings (Hatfield and Kojima, 2010, Theorem 1 and Aygün and Sönmez, 2012, Theorem 1). A weaker condition, weak substitutability, is necessary to guarantee the existence of stable matchings for all possible "unit-demand" choice functions or preferences of other agents (Hatfield and Kojima, 2008, Proposition 1).[3] That is, if a firm's choice function does not satisfy weak substitutability, then there are "unit-demand" choice functions for

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[1] See Roth (2002) for a comparison between real-life mechanisms that produce stable matchings and real-life mechanisms that produce unstable matchings.

[2] We refer to agents on the "one" side of the market as firms and to agents on the "many" side of the market as workers, although the applications of the many-to-one matching with contracts model are not restricted to labor markets. For example, the model has been applied to the analysis of school choice with soft-caps by Hafalir et al. (2013), cadet-branch matching by Sönmez and Switzer (2013) and matching under distributional constraints by Kamada and Kojima (2015).

[3] A firm's choice function satisfies unit-demand if it selects at most one contract from any given set of contracts. This can be the case, for example, when a firm has a single position. Unit-demand implies bilateral substitutability.

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other firms and preferences for workers such that no stable matching exists.

Our main result states that if a choice function satisfies the IRC condition and selects at most two contracts from any given set of contracts (two-unit demand), then weak substitutability implies bilateral substitutability. Hence, for any choice function that satisfies the IRC condition and two-unit demand, weak substitutability and bilateral substitutability are equivalent. An implication of this equivalence is that, provided that each firm's choice function satisfies the IRC condition and two-unit demand, bilateral substitutability is a maximal domain for the existence of stable matchings.[4]

We elaborate on the implications of our result for matching markets with couples. A prototypical example of a matching market with couples is the market for medical residency positions in the U.S. In this market, each doctor can apply to hospitals as single or as part of a couple, and each hospital can have multiple positions (see Roth, 2008, for more details). Hatfield and Kojima (2010) observe that the market for medical residency positions is an instance of a many-to-many matching with contracts model where there are two contracts between each couple and each hospital, one for each member of the couple. We refer to this model of the market for medical residency positions as the "couples model".[5]

In the couples model, and more generally in many-to-many matching, several stability concepts have been proposed depending on what types of blocking coalitions are allowed. Such stability concepts reduce to the usual stability concept when each hospital has a single position, and there are no obvious reasons to focus on one particular stability concept (Kojima et al., 2013, discuss this issue in the context of the couples model). The couples model allows each hospital to have complex preferences over subsets of doctors. However, in applications hospitals often have preferences with a simple structure: the rank of a doctor at a given hospital is independent of her colleagues. In this case, no generality is lost by treating each hospital with multiple positions as multiple hospitals with a single position each, although this approach would lead to a particular stability concept (Kojima et al., 2013, footnote 22). We refer to the model of the market for medical residency positions where each hospital with multiple positions is treated as multiple hospitals with one position each as "the unit-capacity couples model".

In many-to-many matching with contracts (and therefore in the couples model), the substitutability of each agent's choice function is a necessary and sufficient condition for the existence of matchings that satisfy a certain stability concept (Hatfield and Kominers, forthcoming). However, in the unit-capacity couples model substitutability is not necessary (Hatfield and Kojima, 2008). An implication of our equivalence result is that in the unit-capacity couples model, bilateral substitutability is both necessary and sufficient, i.e., a maximal domain, for the existence of stable matchings.

The identification of bilateral substitutability as a new maximal domain is important because until now the only maximal domain known for the unit-capacity couples model was the domain of weakly responsive preferences under a restricted unemployment aversion condition (Klaus and Klijn, 2005 and Klaus et al., 2009).[6] Our result improves upon Klaus and Klijn's (2005) and Klaus et al. (2009) maximal domain result in two ways. First, it does not require the restricted unemployment aversion condition, which is implausible in several situations. For example, it is violated when a couple prefers a high paying job for one of its members and the unemployment for the other member better than two geographically distant jobs. Second, bilateral substitutability is a strictly larger domain than weak responsiveness (Hatfield and Kojima, 2010, Theorem 2 and Example 4).[7]

Finally, we note that the applicability of our results is not restricted to matching with couples markets. Another application is, for example, the higher education scheme in Hungary, where students typically apply for pairs of M.Sc. studies and hence act like couples in the market for medical residency positions (see Biró, 2008, for more details).

2. Notation

To present our results, we only need the following partial description of the matching with contracts model of Hatfield and Milgrom (2005).

There is a single **firm** f and there are (finite and disjoint) sets W of **workers** and X of **contracts**. Each contract $x \in X$ is associated with f and with a worker $x_W \in W$. Let $Y \subseteq X$, we define $Y_W \equiv \cup_{y \in Y}\{y_W\}$ to be the set of workers with contracts in Y.

Given a set of contracts $Y \subseteq X$, f's **choice set** $\mathtt{C}(Y)$ is a subset of Y, i.e., $\mathtt{C}(Y) \subseteq Y$. We assume that f can sign only one contract with any given worker, i.e.,

$$\forall Y \subseteq X,\ \forall x, x' \in \mathtt{C}(Y), \quad x \neq x' \Longrightarrow x_W \neq x'_W.$$

Let $Y \subseteq X$, we define f's **rejected set** of contracts as $\mathtt{R}(Y) \equiv Y \setminus \mathtt{C}(Y)$. We refer to the function that maps each set of contracts to the choice (rejected) set as the **choice (rejection) function**.

3. Conditions on choice functions

A choice function satisfies the irrelevance of rejected contracts condition (IRC) (Aygün and Sönmez, 2013) if the removal of rejected contracts does not affect the choice set. The IRC condition is a mild consistency requirement. In particular, it is easy to check that any choice function generated by the maximization of a strict preference relation satisfies the IRC condition.

Irrelevance of rejected contracts (IRC): $\forall X', X'' \subseteq X$,

$$\mathtt{C}(X'') \subseteq X' \subseteq X'' \implies \mathtt{C}(X') = \mathtt{C}(X'').$$

A choice function satisfies two-unit demand if it never selects sets of size larger than two.

Two-unit demand (TUD): $\forall Y \subseteq X,\ |\mathtt{C}(Y)| \leq 2$.

A choice function satisfies bilateral substitutability (Hatfield and Kojima, 2010) if whenever a contract z is rejected when all available contracts involve different workers, contract z is still rejected when contracts with new workers are added to the choice set.

[4] In the matching literature, a domain $\mathcal{D}$ of individual agents' choice functions (or preferences) is called a maximal domain for the existence of stable matchings if (i) if all agents' choice functions belong to $\mathcal{D}$, then a stable matching exists and (ii) if the choice function of one agent does not belong to $\mathcal{D}$, then there are choice functions for all other agents that belong to $\mathcal{D}$ such that no stable matching exists.

[5] In the couples model, each couple signs at most one contract for each of its members. Therefore, each couple's choice function satisfies two-unit demand.

[6] A couple's preferences are weakly responsive if there exist individual preferences for each of its members such that an improvement in one couple member's job (according to that member's individual preferences) is an improvement for the couple as well. A couple's preferences satisfy restricted unemployment aversion if the couple is worse off when one of his members loses a position that is "acceptable" according to that member's individual preferences.

[7] Weak responsiveness (and hence bilateral substitutability) does not ensure that the set of stable matchings forms a lattice nor that there is a stable and strategy proof mechanism (Klaus and Klijn, 2005).