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A unit root test against globally stationary ESTAR models when local condition is non-stationary☆

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HIGHLIGHTS

- ESTAR model with the local-explosive characteristics is considered.
- A modified Wald-type test is proposed to tackle a nonstandard testing problem.
- The asymptotic distribution of our test statistic is derived.
- Simulation results show that our test performs well.

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ABSTRACT

This paper focuses on testing for the unit root hypothesis against local-explosive or local unit root but globally stationary ESTAR process. A modified Wald-type test for a joint hypothesis where one parameter is one-sided while the others are two-sided under the alternative is proposed. The asymptotic distribution of the test statistic is derived, which is shown to be a function of Brownian motions and does not depend on nuisance parameters. Critical values of the test are tabulated and some simulation results are reported. Results show that the modified Wald-type test performs well.

1. Introduction

Nonlinear time series models like smooth transition autoregressive (STAR) models have recently gained popularity for the modeling of the dynamics of financial and economic data since Teräsvirta (1994). The STAR models can realistically describe the state transition and structural changes continuously, so they are widely used in the fields of industrial output, real exchange rate, unemployment rate and so on (see, Lundbergh and Teräsvirta, 2001, Van Dijk et al., 2002). Existing tests for a unit root against STAR models, see for example Kapetanios et al. (2003) and Teräsvirta et al. (2010), always assume that the time series under investigation has a local-stationary or local unit root process. However, some empirical studies report that many time series under investigation present local-explosive characteristics based on nonlinear STAR model, see for example Baharumshah and Liew (2006) and Ubilava and Helmers (2013). Local-explosive behavior has received a great deal of attention in the analysis of financial time series as it has been successfully applied to a variety of financial time series. Therefore, it is important to take the characteristic into account.

Regarding the ESTAR process, a popular Dickey–Fuller-type test has been proposed by Kapetanios et al. (2003). This test assumes that local parameter α is equals to zero that means the process has a local unit root. However, if the DGP is a local-explosive ESTAR process, using the KSS test may increase the probability of Type II error, since the difference between locally explosive term and local unit root term will have an impact on the KSS test

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statistic, which may suggest that the coefficient on the cube of y_{t-1} is not significant and the null hypothesis is accepted. In the paper, we focus on not only the smoothness parameter but also the local parameter. When we investigate unit root test against ESTAR model with the local-non-stationary characteristics, we are faced with a nonstandard testing problem, i.e. a joint hypothesis where one parameter is one-sided under the alternative while the others are two-sided. Since standard test techniques are not appropriate in this situation, we make use of the approach by Abadir and Distaso (2007) who propose a class of modified test statistics in order to tackle such non-standard testing problems.

In this paper, we investigate the locally non-stationary but globally stationary ESTAR processes. A joint hypothesis where one parameter is one-sided while the others are two-sided under the alternative is considered and we propose a modified Wald-type test for a unit root against an alternative of nonlinear ESTAR process. Furthermore, the asymptotic distribution of the test statistic is derived, which does not depend on nuisance parameters. Some simulation results are reported to test the effect of the statistic. The remainder of this paper is organized as follows. In Section 2 we propose the test statistic, and derive the asymptotic distribution. Section 3 provides asymptotic critical values of the test statistic and addresses the issue of the small sample performance of the proposed test by Monte Carlo simulation. Some conclusion is drawn in Section 4.

2. Asymptotic distribution of the modified Wald statistic

Consider an exponential smooth transition autoregressive of order 1, i.e. ESTAR(1) model,

$$\Delta y_t = \alpha y_{t-1} + \gamma y_{t-1}(1 - \exp\{-\theta(y_{t-1} - c)^2\}) + \varepsilon_t, \theta > 0 \quad (2.1)$$

where ε_t is a martingale difference sequence with conditional variance σ^2 and $\sup_t E(\sigma^4) < \infty$. In the Eq. (2.1), α, γ, θ and c are unknown parameters. Here we can assume that y_t is a mean zero stochastic process. When the process y_t has nonzero mean and/or a linear time trend, Kapetanios et al. (2003) suggest to demean or detrend the data. To be specific, for the case with nonzero mean and nonzero linear trend, i.e., where $x_t = l'd_t + y_t$ with $d_t = 1$ or $d_t = [1\ t]'$ and l is a coefficient vector, we use the demeaned and detrended data $y_t = x_t - \hat{l}'d_t$, where $\hat{l}$ is the OLS estimator of l and unit root test is applied to y_t.

Unit root test against ESTAR alternative is complicated because of the nuisance parameters. A popular approach to avoid the presence of nuisance parameters under the null hypothesis H_0 : $\theta = 0$ is to use a suitable Taylor approximation of the smooth transition function $G(y_{t-1}; \theta, c) = 1 - \exp\{-\theta(y_{t-1} - c)^2\}$ around $\theta = 0$. The auxiliary regression equation based on Kiliç (2004) is given by

$$\Delta y_t = \beta_1 y_{t-1} + \beta_2 y_{t-1}^2 + \beta_3 y_{t-1}^3 + u_t. \quad (2.2)$$

Our interest in the paper centers on the unit root test against the local-explosive or local unit root but globally stationary ESTAR process. That means, the problem of interest is how to test the process is a random walk or a local-non-stationary but global-stationary ESTAR model. Kapetanios et al. (2003) show that the process is globally stationary if $|1 + \alpha + \gamma| < 1$. If the process is local-explosive ($\alpha > 0$) or local unit root ($\alpha = 0$), we can conclude that the parameter γ must be less than zero provided that the process is globally stationary. Hence, β_3 in auxiliary regression equation (2.2) is less than zero while β_1 and β_2 may take negative or positive values by Taylor approximation which are dependent of the location parameter c.

Here we are interested in the pair of hypotheses given by H_0 : $\beta_1 = \beta_2 = \beta_3 = 0$ against the alternative H_1 : $\beta_1 \neq 0, \beta_2 \neq 0, \beta_3 < 0$. Note that the partially two-sided nature of the alternative stems from the fact β_1 and β_2 are allowed to take real values. This testing problem is nonstandard in the sense that two parameters are two-sided under H_1 while the another is one-sided. A standard Wald test would be inappropriate and we apply the methods of Abadir and Distaso (2007) to derive a suitable test. The modified Wald test builds upon the one-sided parameter β_3 and the transformed two-sided parameters that are stochastically independent by definition.

Next, we propose the modified Wald-type statistic to test the hypotheses. Let $\beta = (\beta_1\ \beta_2\ \beta_3)'$. Following the notation of Abadir and Distaso (2007), the null hypothesis can be represented by

$$H_0 : h(\beta) = (h_l(\beta)'\ h_3(\beta))' = (0\ 0\ 0)',$$

against

$$H_1 : h_l(\beta) \neq 0 \quad \text{or} \quad h_3(\beta) < 0,$$

where $h_l(\beta)$ is 2×1 matrix. The standard Wald test statistic based on the Hessian matrix $\mathscr{H}$ is

$$W = h(\hat{\beta})'\hat{V}^{-1}h(\hat{\beta}),$$

where 3×3 matrix $\hat{V} = \frac{\partial h(\beta)}{\partial \beta'}\Big|_{\beta=\hat{\beta}} (-\hat{\mathscr{H}})^{-1} \frac{\partial h(\beta)'}{\partial \beta}\Big|_{\beta=\hat{\beta}}$ with elements $\hat{v}_{ij}$. And here $\hat{\beta}$ is the OLS estimate of β. Using block-matrix, we have $\hat{V} = \begin{pmatrix} \hat{V}_l & \hat{V}_{l3} \\ \hat{V}_{l3}' & \hat{v}_{33} \end{pmatrix}$, where $\hat{V}_l = \begin{pmatrix} \hat{v}_{11} & \hat{v}_{12} \\ \hat{v}_{12} & \hat{v}_{22} \end{pmatrix}$ and $\hat{V}_{l3} = \begin{pmatrix} \hat{v}_{13} & \hat{v}_{23} \end{pmatrix}'$, so the test statistic can be rewritten quadratic forms as

$$W = h_{l\cdot 3}(\hat{\beta})'\hat{V}_{l\cdot 3}^{-1}h_{l\cdot 3}(\hat{\beta}) + h_3(\hat{\beta})^2\hat{v}_{33}^{-1},$$

where $h_{l\cdot 3}(\hat{\beta}) = h_l(\hat{\beta}) - \frac{h_3(\hat{\beta})\hat{V}_{l3}}{\hat{v}_{33}}$, and $\hat{V}_{l\cdot 3} = \hat{V}_l - \frac{\hat{V}_{l3}\hat{V}_{l3}'}{\hat{v}_{33}}$, which is an estimate of the asymptotic variance of $h_{l\cdot 3}(\hat{\beta})$. The modified Wald test statistic of Abadir and Distaso (2007) is given by

$$\tau = h_{l\cdot 3}(\hat{\beta})'\hat{V}_{l\cdot 3}^{-1}h_{l\cdot 3}(\hat{\beta}) + 1_{h_3(\hat{\beta})<0}h_3(\hat{\beta})^2\hat{v}_{33}^{-1},$$

where 1_A is an indicative function of set A. Let $h_l(\beta)$ be $(\beta_1\ \beta_2)'$ and $h_3(\beta)$ be β_3, we can get the test statistic

$$\tau = \begin{pmatrix} \hat{\beta}_1 - \frac{\hat{\beta}_3\hat{v}_{13}}{\hat{v}_{33}} & \hat{\beta}_2 - \frac{\hat{\beta}_3\hat{v}_{23}}{\hat{v}_{33}} \end{pmatrix} \times \begin{pmatrix} \hat{v}_{11} - \frac{\hat{v}_{13}^2}{\hat{v}_{33}} & \hat{v}_{12} - \frac{\hat{v}_{13}\hat{v}_{23}}{\hat{v}_{33}} \\ \hat{v}_{12} - \frac{\hat{v}_{13}\hat{v}_{23}}{\hat{v}_{33}} & \hat{v}_{22} - \frac{\hat{v}_{23}^2}{\hat{v}_{33}} \end{pmatrix}^{-1} \times \begin{pmatrix} \hat{\beta}_1 - \frac{\hat{\beta}_3\hat{v}_{13}}{\hat{v}_{33}} & \hat{\beta}_2 - \frac{\hat{\beta}_3\hat{v}_{23}}{\hat{v}_{33}} \end{pmatrix}' + 1_{\hat{\beta}_3<0}\frac{\hat{\beta}_3}{\hat{v}_{33}}.$$

Based on the auxiliary regression equation (2.2), we can obtain $\hat{\beta} = \left(X'X\right)^{-1}X'U$ and $\hat{V} = \sigma_T^2\left(X'X\right)^{-1}$ (see Appendix). A simpler and more intuitive way to formulate this statistic is

$$\tau = \tau_l^2 + 1_{\tau_3<0}\tau_3^2.$$

The two summands appearing in the test statistic τ can be interpreted as follows: The component τ_3^2 is a squared t-statistic for the hypothesis $\beta_3 = 0$. And the first term τ_l^2 is a squared t-statistic for the hypothesis $(\beta_1\ \beta_2) = (0\ 0)$ being orthogonal to β_3. Next, the limiting distribution of τ is derived. All the proofs are given in the Appendix.

Theorem 1. *Suppose y_t is a random walk with $y_0 = 0$, then*

$$\tau \Rightarrow \mathscr{A}(W(r)) + \mathscr{B}(W(r)),$$