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Detecting structural changes under nonstationary volatility

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HIGHLIGHTS

- The U-statistic proposed by Juhl and Xiao (2013) can be used to test against structural changes under nonstationary volatility.
- The test allows for conditional heteroskedasticity and time-varying unconditional variance, and can detect any smooth or abrupt structural changes.
- We advocate using a bootstrap method to improve the size performance in finite samples.

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ABSTRACT

This paper shows that the U-statistic for moment condition stability proposed by Juhl and Xiao (2013) can be used to test against structural changes in regression coefficients under nonstationary volatility. We investigate the power property under the alternative, and prove that the test is consistent against single break, multiple breaks and smooth structural changes. Finally, we advocate using a bootstrap method to improve its size performance in finite samples.

1. Introduction

Detection of structural changes in economic relationships is a long-standing problem in econometrics. Up to now, most existing tests for parameter stability are constructed under the assumption that the unconditional volatility stays constant, see Ploberger and Krämer (1992), Andrews and Ploberger (1994), Bai and Perron (1998), and Chen and Hong (2012). While this sort of setup is simple and can make the testing procedures maintain much of its parsimonious structure. However, the inference is possibly misleading if the volatility is nonstationary. Theoretically speaking, parameter instability resulting from change in distribution is naturally accompanied by instability in volatility, which has been documented by many empirical studies, see Mikosch and Stărică (2004), and Justiniano and Primiceri (2008). Hence, it is highly desirable to develop tests that are also valid in the presence of nonstationary volatility. To our knowledge, there are four tests designed for nonstationary volatility. However, Pitarakis (2004), Perron and Zhou (2008) and Xu (2015) only allow for one single break in regression coefficients, and the critical values in Xu (2015) are obtained only through Monte-Carlo simulations. Kristensen's (2012) nonparametric tests can detect smooth changes, but are not able to consistently detect abrupt breaks. Furthermore, the error term in the tests does not allow for conditional heteroskedasticity such as GARCH effects.

In this paper, we show that the nonparametric test of moment condition stability proposed by Juhl and Xiao (2013) can be used to test against structural changes in regression coefficients under nonstationary volatility. Moreover, the test can be applied to the case where nonstationary unconditional volatility and conditional heteroskedasticity coexist in error terms. We investigate the power property of the test under the alternative, and prove that the test is consistent against single break, multiple breaks and smooth structural changes. In order to reduce under-sized distortion and sensitivity of bandwidth choice, we advocate using a wild bootstrap method to improve the size performance in finite samples.

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2. Test and assumptions

We consider the following time-varying coefficient model

$$Y_t = X_t'\alpha_t + u_t, \quad t = 1, \ldots, T \tag{1}$$

where Y_t is a dependent variable, X_t is a $d \times 1$ vector of explanatory variables and α_t is a $d \times 1$ possibly time-varying parameter vector. The regressor vector X_t can contain exogenous explanatory variables and lagged dependent variables. Thus, both static and dynamic regression models are covered. The error is decomposed as

$$u_t = \sigma_t \varepsilon_t. \tag{2}$$

The process $\{\varepsilon_t\}_{t=1}^T$ is a martingale difference sequence that satisfies $E(\varepsilon_t|F_{t-1}) = 0$ and $E(\varepsilon_t^2|F_{t-1}) = h(F_{t-1})$, where $F_{t-1} = \{X_t', X_{t-1}', \ldots, \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots\}$ and $h(F_{t-1})$ is the conditional heteroskedasticity of ε_t, while the process $\left\{\sigma_t^2\right\}_{t=1}^T$ is modeled as a time-varying trend that functions as a proxy for all factors that affect the unconditional volatility.

We are interested in testing the constancy of the regression parameter in (1). The null hypothesis of interest is

$$H_0 : \alpha_t = \alpha \quad \text{for all } t \tag{3}$$

and the alternative hypothesis is

$$H_A : H_0 \quad \text{is false.} \tag{4}$$

Define $m_t = X_t\left(Y_t - X_t'\alpha\right)$, then we have $E(m_t|F_{t-1}) = 0$ for all t under the null hypothesis; otherwise we have $E(m_t|F_{t-1}) = X_tX_t'(\alpha_t - \alpha) \neq 0$ due to the instability of α_t. Hence, testing the stability of α_t is equivalent to checking whether the conditional moment $E(m_t|F_{t-1}) = 0$ holds over time. Under the null hypothesis, the estimator of $\{m_t\}_{t=1}^T$ can be represented by $\left\{\hat{m}_t = X_t\hat{u}_t\right\}_{t=1}^T$, where $\left\{\hat{u}_t\right\}_{t=1}^T$ is the OLS residuals from the regression of Y_t on X_t over the full sample.

Applying Juhl and Xiao (2013)'s U-statistic for moment condition stability, we consider the following test statistic,

$$\hat{\lambda}_T = \frac{1}{T^2h}\sum_{t=1}^T\sum_{s\neq t}K_{s,t}\hat{m}_t'\hat{m}_s \tag{5}$$

where $K_{s,t} = K\left(\frac{s-t}{Th}\right)$ is the kernel function and h is a bandwidth parameter. If the null hypothesis is true, the U-statistic $\hat{\lambda}_T$ should be close to zero, and asymptotically normally distributed. Under the alternative, it will be distant away from zero. In order to obtain a valid test, $\hat{\lambda}_T$ needs to be standardized by $Th^{1/2}$ and a variance estimator, say, $\hat{\varphi}^2$, to achieve standard normal limit. Now, we construct the following test statistic:

$$\hat{U}_T = \frac{Th^{1/2}\hat{\lambda}_T}{\hat{\varphi}} \tag{6}$$

where

$$\hat{\varphi}^2 = \frac{2}{T^2h}\sum_{t=1}^T\sum_{s\neq t}K_{s,t}^2\left(\hat{m}_t'\hat{m}_s\right)^2. \tag{7}$$

To derive the asymptotic normality of the above test, we introduce the following regularity conditions.

Assumption 1. (i) $\{X_t', \varepsilon_t\}'$ is a $(d+1) \times 1$ *β-mixing* process with mixing coefficients $\{\beta(j)\}_{j=1}^\infty$ satisfying $\sum_{j=1}^\infty j^2\beta(j)^{\delta/(1+\delta)} < C$ for some $0 < \delta < 1$; (ii) $M = E\left(X_tX_t'\right)$ and $\Omega = E\left(X_tX_t'\varepsilon_t^2\right)$; (iii) $E\left(X_{it}^8\right) < C$ for $i = 1, \ldots, d$, and $E\left(\varepsilon_t^8\right) < C$.

Assumption 2. $\{\varepsilon_t\}$ is a martingale difference sequence (m.d.s) such that $E(\varepsilon_t|F_{t-1}) = 0$, and $\mathrm{Var}(\varepsilon_t) = 1$, *where* $F_{t-1} = \{X_t', X_{t-1}', \ldots, \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots\}$.

Assumption 3. $\sigma_t^2 = \sigma^2(t/T)$ is a bounded non-negative function, and has continuous second derivatives except for a finite number of points on [0, 1].

Assumption 4. $\hat{\alpha}$ is a parameter estimator such that $\sqrt{T}\left(\hat{\alpha} - \alpha^*\right) = O_p(1)$, where $\alpha^* = p\lim_{T\to\infty}\hat{\alpha}$ and $\alpha^* = \alpha$ under H_0.

Assumption 5. The kernel $K(\cdot)$ is a symmetric and bounded probability density function with support on $[-1, 1]$, satisfying $K(0) \geq K(r)$ for all r, $\int_{-1}^1 K(r)dr = 1$ and $\int_{-1}^1 K(r)^2dr < \infty$.

Assumption 6. As $T \to \infty$, $h \to 0$ and $Th^2 \to \infty$.

The *β-mixing* condition in Assumption 1 puts some restrictions on the temporal dependence in $\{X_t', u_t\}$. $E(\varepsilon_t|F_{t-1}) = 0$ in Assumption 2 implies that the linear regression model is correctly specified under H_0. Additionally, $\mathrm{Var}(\varepsilon_t) = 1$ in Assumption 2 allows for the conditional variance of ε_t to be heteroskedastic. Assumption 3 covers both smooth structural changes and abrupt structural breaks with known or unknown breakpoints in σ_t^2. Assumption 4 holds for any $\sqrt{T}$-consistent estimator for α under H_0. We allow but not restricted to the OLS estimator $\hat{\alpha}$. Assumptions 5 and 6 are standard assumptions in the kernel regression literature.

Theorem 1. *Suppose that Assumptions 1–6 hold, then under the null hypothesis of $\alpha_t = \alpha$ we have*

$$\hat{U}_T \xrightarrow{d} N(0, 1) \tag{8}$$

where $\hat{\varphi}^2$ is a consistent estimator of $\varphi^2 = 2trace(\Omega\Omega)\int_{-1}^1 K^2(v)dv\int_0^1 \sigma^4(r)dr$ as $T \to \infty$.

Proof. See in the Supplementary Material (see Appendix A).

Remark. This theorem is obtained by directly using the result of Theorem 3.1 in Juhl and Xiao (2013). Because $\hat{U}_T$ converges to a standardized normal distribution, one can implement the test for H_0 by comparing it with a $N(0, 1)$ critical value. Additionally, the estimation error in $\hat{\alpha}$ is negligible asymptotically and has no impact on $\hat{U}_T$ because it converges at rate $\sqrt{T}$, which is faster than the rate of $\hat{U}_T$ approaching H_0 under local alternatives (see Theorem 3).

To study the asymptotic power of the test under H_A, we rewrite $\alpha_t = \alpha\left(\frac{t}{T}\right)$ and impose the following assumption:

Assumption 7. $\alpha\left(\frac{t}{T}\right)$ is a bounded function, and has continuous second derivatives except for a finite number of points on [0, 1].

In other words, we permit $\alpha\left(\frac{t}{T}\right)$ to have many finite discontinuities. Hence, one single break or multiple breaks can be regarded as special cases of our model. As a result, the test can be used to detect abrupt or smooth structural changes.

Theorem 2. *Suppose that Assumptions 1–7 hold, then for any sequence of nonstochastic constants $\left\{C_T = o\left(T\sqrt{h}\right)\right\}$ we have*

$$\Pr(\hat{U}_T > C_T) \to 1 \tag{9}$$

under H_A as $T \to \infty$.