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Serial dictatorship and unmatch reduction: A problem of Japan's nursery school choice

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HIGHLIGHTS

- The trade-off between using serial dictatorship and unmatch reduction in school choices is analyzed.
- The result depends on conditions such as children's preferences and schools' capacities.
- The study is motivated by a social problem in Japan called the "*taiki-jidou* (waiting children)" problem.

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ABSTRACT

We conduct a simulation analysis to see the trade-off between employing serial dictatorship and reducing unmatched children in nursery school choices. It is motivated by a social problem in Japan: there are a large number of children who wish to get into nursery schools but stay unmatched, while serial dictatorship is widely used.

1. Introduction

In Japan, each municipality conducts a centralized matching between nursery schools and children. The (parents of the) children represent their preferences and are ranked by the local authority based on the priority for taking the childcare services (determined by e.g. parents' working time per week), and then the matching is conducted. While the matching mechanism depends on the municipality, the most prevailing one in the 23 wards in Tokyo is serial dictatorship (hereafter SD), and only a few wards use Boston or some other mechanism (Sasaki, unpublished). In SD, the child with the highest rank is assigned to her top choice, the child with the next highest rank is assigned to her top choice among the remaining seats, and so on.[1]

SD is efficient and eliminates justified envy, but does not care about unmatches. Consider the following example. There are two children (1 and 2) and two nursery schools (A and B). For 1, A is preferable to B and both schools are acceptable, while only A is acceptable for 2. 1 is ranked higher than 2. Each school offers only one seat. Then SD selects a matching, where 1 is matched with A, while 2 is unmatched. If we impose only individual rationality as the minimal requirement, another matching is possible: 1 and 2 are matched with B and A, respectively. This does not eliminate justified envy but results in less unmatches. Thus there is a trade-off between employing SD and unmatch reduction.

In this note, we conduct a simulation analysis to see the trade-off by comparing the number of matched children by SD with that obtained by unmatch minimization subject to individual rationality. The result shows that the level of the trade-off depends on some conditions such as preferences of children and capacities of schools. One may think imposing only individual rationality in the latter case is too weak. We also show that the same level of unmatches can be achieved even when we additionally impose

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[1] It is often called simple SD in the literature.

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efficiency and moreover require everyone matched under SD to be assigned to some school. (The example above satisfies these conditions.)

The analysis is motivated by a social problem in today's Japan, so-called the "*taiki-jido* (waiting children)" problem. Particularly in urban cities, there are a large number of children who wish to get into nursery schools but stay unmatched, and the number has been increasing. In most existing applications of market design such as medical labor markets (Roth, 1984) and school choices (Abdulkadiroglu and Sönmez, 2003), one usually does not care about unmatching when studying desirable mechanisms. But, in Japan's nursery school choices, unmatch reduction can be one important objective for market designers as it is the society's requirement.[2]

2. Formulation

A nursery school choice can be formulated as a student placement problem by Balinski and Sönmez (1999). Let S and C be the finite sets of nursery schools and children, respectively. Each child c has a strict preference order on $S \cup \{c\}$ denoted by $\succ_c$. When $s \succ_c s'$, c prefers s to s'. c means staying unmatched, and if $s \succ_c c$, s is acceptable for c. Let $r_c \in \mathbb{R}$ be c's rank. When $r_c > r_{c'}$, c is ranked higher than another child c'. The ranking is assumed to be strict. Let q_s be the capacity of school s. Then the problem is defined by $(S, C, \{\succ_c\}_{c \in C}, (r_c)_{c \in C}, (q_s)_{s \in S})$.[3]

A matching is function $\mu : C \to S \cup C$ satisfying the following two properties. First, for all $c \in C$, $\mu(c) \in S \cup \{c\}$. $\mu(c)$ is the school to which c is assigned. If $\mu(c) = c$, c is unmatched. Second, for all $s \in S$, $|\mu^{-1}(s)| \leq q_s$. A mechanism is a function from a school choice problem to a matching. Matching μ is *individually rational* if, for all $c \in C$, $\mu(c) \succ_c c$ or $\mu(c) = c$. μ is *non-wasteful* if, for all $c \in C$, $|\mu^{-1}(s)| = q_s$ for any $s \in S$ such that $s \succ_c \mu(c)$. μ *eliminates justified envy* if, for all $s \in S$ and $c \in C$ such that $s \succ_c \mu(c)$, $r_{c'} > r_c$ for any $c' \in \mu^{-1}(s)$. For two matchings μ and μ', if there is no $c \in C$ such that $\mu(c) \succ_c \mu'(c)$ and there exists $c' \in C$ such that $\mu'(c') \succ_{c'} \mu(c')$, then μ' Pareto dominates μ. If μ is not Pareto dominated by any other matchings, it is *efficient*. A mechanism satisfies individually rationality, non-wastefulness, eliminating justified envy or efficiency if it always selects a matching satisfying each property. A mechanism is *strategy-proof* if it is always optimal for every child to represent the true preference.

SD is a mechanism described as follows. First, the child with the highest rank is assigned to her most preferable nursery school among the acceptable ones with remaining capacities. If there are no such schools, then she stays unmatched. Second, the child with the second highest rank is assigned in the same way. Then the other children are assigned similarly in the order of the ranking.

SD is the only mechanism that is efficient and eliminates justified envy (Balinski and Sönmez, 1999). Note that efficiency implies individual rationality and non-wastefulness. It is also strategy-proof.

3. Simulation analysis

3.1. Method

To see the trade-off of employing SD and unmatch reduction, we compare the number of matched children by SD with that obtained by unmatch minimization subject to individual rationality (hereafter UM). For any school choice problem, the latter is calculated by solving the following binary integer programming problem.

$$\begin{aligned}\text{maximize} \quad & \sum_{s \in S} \sum_{c \in C} \delta_{sc} x_{sc} \\ \text{s.t.} \quad & \text{(i) } \forall s \in S, \sum_{c \in C} x_{sc} \in [0, q_s] \\ & \text{(ii) } \forall c \in C, \sum_{s \in S} x_{sc} \in [0, 1] \\ & \text{(iii) } \forall s \in S, \forall c \in C, x_{sc} \in \{0, 1\}.\end{aligned}$$

For $s \in S$ and $c \in C$, $x_{sc} = 1$ if c is matched with s, otherwise $x_{sc} = 0$. δ_{sc} is 1 if s is acceptable for c, otherwise it is $-\infty$. Its solution gives a matching with minimal unmatches under individual rationality, and the optimized objective function tells the number of matched children there. Note that this is not a mechanism since the problem may have multiple solutions, though any solution gives minimal unmatches. Then we have the next proposition. (The proof is shown in the Appendix.)

Proposition 1. *For any school choice problem, there exists a matching satisfying the following three properties:* (i) *the number of matched children is equal to that under UM,* (ii) *every child who is matched under SD is assigned to some school, and* (iii) *efficient.*

Thus, even if we additionally impose properties (ii) and (iii), there is a matching that can achieve the minimal unmatch number. In other words, there always exists a solution of UM that satisfies these conditions. (Note that efficiency implies individual rationality.)

In what follows, we compare the numbers of matched children in SD and UM. If the difference is negligibly small, we can conclude SD is desirable also in terms of unmatch reduction. Otherwise we face the trade-off. In this case, by giving up the use of SD, we may be able to achieve some level of unmatch reduction even when we require the properties of the proposition above. We emphasize that our intention to study UM is not to propose it as a promising alternative mechanism to SD (indeed UM is not a mechanism) but to use it just as a benchmark to see the trade-off. Thus we use true preferences in the calculation of UM.

The simulation setting is as follows. $|S| = 20$ and $|C| = 200$. The rank is generated randomly. For simplicity, every nursery school has the same capacity, say q. We generate preferences in the following manner. For each school s and child c, let $u_c(s)$ be c's utility of s, which is defined by:

$$u_c(s) = \alpha v(s) + (1 - \alpha) v_c(s),$$

where $v(s)$ and $v_c(s)$ are the common evaluation of s among the children (e.g. its basic service level) and c's personal evaluation of s (e.g. the distance from home), respectively. $v(s)$, $v_c(s) \in [0, 100]$ and they are generated randomly. $\alpha \in [0, 1]$ is the correlation parameter: the bigger α is, the more correlated their preferences are. Then suppose only top β schools in terms of the utility are acceptable for the child. For simplicity, we assume it is common among everyone. We prepare independently 30 sets of $v(s)$ and

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[2] More precisely, in the most competitive cities, where the number of the applicants is about twice the total capacity, the schools' capacities are completely (or almost completely) fulfilled even by SD. In these cities, there is no trade-off. However, in many other cities, there remain a certain amount of seats in a resulting matching, whereas there are children unmatched. Usually this is not the final result, and the second (or third, if necessary) matching is conducted in some way between unmatched children and schools with remaining seats. But this requires the local authority some expense (which usually increases when unmatched children increase). Also, children matched in the second round have to be assigned to schools that were unacceptable for them in the first round (as long as the first matching is non-wasteful). For these reasons, there is a requirement for unmatch reduction in the first matching process (according to our interview with a municipal government operator). Thus we consider the discussion in this note can potentially be beneficial for these cities. (These situations are as of 2016.)

[3] It is a special case of college admission problems (Gale and Shapley, 1962) where every school has the same preference. Then a matching is individually rational, non-wasteful and eliminates justified envy if and only if it is stable. In this case, SD is equivalent to the deferred acceptance algorithm (Morrill, 2013).