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Sustainable growth with irreversible stock effects of renewable resources

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HIGHLIGHTS

- Stock effects of pollution is examined in a model of sustainable growth.
- Irreversible threshold of renewable resource stock is considered.
- Maximum critical level of initial emissions for sustainability is derived.
- Sustainability is feasible via pollution tax depending on the initial emissions.

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ABSTRACT

This paper examines sustainability conditions when the stock effects of renewable resources prevail, and characterizes the maximal critical level of initial pollution emissions such that Pigovian tax alone ensures sustainable growth.

1. Introduction

López and Yoon (2014) shows that continuous economic growth with eventually decreasing pollution emissions is achievable with Pigovian tax as long as either the consumer or producer's elasticity of substitution is sufficiently large or if the consumer elasticity of marginal utility of income is greater than one in an economy with two final goods and inputs.[2] However, they do not consider stock accumulation effects of pollution and ignore the possibility of irreversible threshold occurring as pollution accumulates in the atmosphere. We develop here a two-sector endogenous growth model explicitly accounting for the existence of irreversible thresholds affecting the stock of renewable natural resources such as the stock of clean air in the atmosphere. We show that for each level of clean air stock, there exists a corresponding critical level of initial pollution emissions such that Pigovian tax alone can still ensure environmentally sustainable economic growth if the initial pollution level lies below the critical level.

2. Framework of the analysis

We consider an economy producing a clean and a dirty good. The dirty good production emits pollution, x, while production of the clean good emits no pollution. The dirty output y_d is modeled by a constant elasticity of substitution:

$$y_d = F(k_d, x) = \left[\alpha k_d^{-\frac{1-\omega}{\omega}} + (1-\alpha)(x)^{-\frac{1-\omega}{\omega}}\right]^{-\frac{\omega}{1-\omega}}, \quad (1)$$

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2 For an earlier pioneering research, see Figueroa and Pasten (2013).

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where k_d is the amount of capital employed; ω is the elasticity of substitution between capital and pollution and α is a fixed distribution coefficient. The output of the clean good, which is assumed to be used as a final good as well as new capital, is modeled by the linear production technology:

$$y_c = A(k - k_d)$$

where A is the return to capital in the clean sector. Let $p \equiv p_d/p_c$ denote the relative price of dirty good. Then the gross capital accumulation, which is equal to net savings (income less consumption), can be expressed in units of the clean good,

$$\dot{k} = A_c(k - k_d) + pA_dF(k_d, x) - c - \delta k, \tag{2}$$

where $\dot{k} \equiv dk/dt$; c is the total consumption expenditure (e.g., $c \equiv c_c + pc_d$); δ denote the net capital accumulation and depreciation rate.

Let us denote the stock of renewable resource (e.g., clean air) in the upper atmosphere as E, the threshold of minimal stock of clean air below which an environmental catastrophe occurs as $\underline{E}$, the pristine stock level by $\overline{E}$. Following Acemoglu et al. (2012), we assume that before an environmental catastrophe occurs, the rate of natural atmospheric purification is given as constant, $0 < \psi < 1$. Thus,

$$\begin{aligned} \dot{E} &= \psi E - x \quad \text{for } \underline{E} \leq E < \overline{E}. \\ &= -x \quad \text{for } E < \underline{E}. \end{aligned} \tag{3}$$

The welfare of the representative consumer is comprised of two parts, a utility derived from the consumption of goods and the disutility generated by pollution. When $E < \overline{E}$, the economy is in catastrophic calamity, and the consumption falls to zero (e.g., Weitzman, 2009). We specify the consumer's total welfare function as[3]:

$$\begin{aligned} U(c, x; E) &\equiv \frac{1}{1-a}\left(\frac{c}{e(1,p)}\right)^{1-a} - \frac{x^{1+\eta}}{1+\eta} \quad \text{when } E \geq \underline{E} \\ &\equiv -\infty \quad \text{when } E < \underline{E}, \end{aligned}$$

where $a > 0$ is a parameter equal to elasticity of marginal utility (EMU) of consumption, and $e(1, p)$ is the unit (dual) expenditure function, given as $e(1, p) = \left[\gamma_c + \gamma_d p^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$, where σ is the consumption elasticity of substitution between the dirty and clean goods, and $\gamma_c > 0$ and $\gamma_d > 0$ are fixed parameter.

Assuming a fixed pure time discount rate (ρ) and socially optimal intervention, the competitive economy is modeled "as if" it maximizes the present discounted value of the utility function:

$$\int_0^\infty U(c, x; E)\exp(-\rho t)dt,$$

subject to the budget constraint (e.g., Eq. (2)), clean air stock level constraint $E \geq \underline{E}$ (Eq. (3)) and the initial conditions $k = k_0$ and $E = E_0$.

Assuming that both goods are always produced (e.g., $k_d(t) < k(t)$), the current value Hamiltonian function is:

$$\begin{aligned} H_E &= U(c, x, E) + \lambda\left[A(k - k_d) + pF(k_d, x) - c - \delta k\right] \\ &\quad + \mu\left[\psi E - x\right] + \phi\left[E - \underline{E}\right] \end{aligned} \tag{4}$$

where λ and μ denote co-state variables each representing the shadow price of man-made capital and natural capital, respectively while $\phi \geq 0$ is a time-varying Lagrange multiplier associated with the stock constraint.

When the stock constraint is *not* binding, the first-order necessary conditions for maximization of (4) and instantaneous market clearing conditions implies the following dynamical systems for $\hat{p}$, $\widehat{\left(\frac{k_d}{x}\right)}$ and $\hat{x}$[4]:

$$\hat{p} = M\left(1 - S_k\right)\left[\eta/a + 1\right]/\left|W\right| > 0, \tag{5}$$

$$\widehat{(k_d/x)} = M\left[\eta/a + 1\right]\omega/\left|W\right| > 0, \tag{6}$$

$$\hat{x} = MV(t)/\left|W\right|, \tag{7}$$

where $V(t) \equiv (1 - s(t) + s(t)S_k(t))\left((1/a) - \sigma\right) + (\sigma - \omega)S_k(t)$ and $|W| \equiv [(1 - S_k(t))(1 + z(t)\eta) + S_k(t)] + \eta S_k(t)\omega > 0$ where $s(p) \equiv \gamma_d/(\gamma_c p^{\sigma-1} + \gamma_d)$ is the consumer's budget share of the dirty final good and $S_k = \alpha\left[(1-\alpha)\left(k_d/x\right)^{\frac{1-\omega}{\omega}} + \alpha\right]^{-1}$ is the cost share of capital in production.

López and Yoon (2014) has shown that the growth rate of real consumption expenditure, which is given as $\widehat{\left(\frac{c}{e}\right)} = \frac{1}{a}\left[M - s(p)\hat{p}\right]$, remains positive throughout the equilibrium dynamic path for any positive ω and σ. Then if there exists a finite time, $T \geq 0$, such that $\hat{x} < 0$, for any $t > T$, and that $E(t) \geq \underline{E}$ for all t, the economy becomes *environmentally sustainable* in a growing economy over time.[5] In an environmentally sustainable economy, we have that $\lim_{t\to\infty}\hat{x} < 0$,

3. Stock effects

Since Eq. (7) gives the (optimal) rate of change of x that depends on the relative magnitude of a, σ, ω, the full path of x is entirely determined by the initial level of pollution emissions, x_0. The question is whether along this path the stock of clean air ever reaches the catastrophic level.

In order to identify such a critical level of initial emissions, we first note that for any given initial level of man-made capital, a unique optimal growth path for p, (k_d/x) and x is derived.[6] In particular, we can define the pollution emissions at a point in time as:

$$x(t) = x_0\int_0^t \exp(g(\upsilon)\upsilon)d\upsilon,$$

where $g(\upsilon)$ is the rate of change of pollution at time υ, which is a function of all parameters and the predetermined variable, k_0. As we show below, the effect of the initial clean air stock on $x(t)$ occurs entirely through its effect on x_0. From Eq. (4)

$$E(t) = \exp(\psi t)\left(E_0 - \int_0^t x(\upsilon)\exp(-\psi\upsilon)d\upsilon\right) \tag{4'}$$

for $E(t) \geq \underline{E}$; E_0 is the initial, predetermined level of the stock of clean air. We can then define the unique path of pollution emission flows and stock of clean air as conditional functions of the (endogenous) initial values of pollution emissions as well as of the (predetermined) initial stock of clean air as follows:

$$x(t) = G(t, x_0; k_0, \chi) \quad \text{and} \quad E(t) = J(t, x_0; k_0, E_0, \chi),$$

[3] As in Weitzman (2009) the consumption utility becomes $-\infty$ for $c = 0$ when $a > 1$. Since it is not possible to describe individual optimization when the economy falls toward the catastrophic calamity, we assume that the environmental damage itself is greater than any finite conceivable magnitude (and hence minus infinity) even when $a < 1$.

[4] See López and Yoon (2014) for derivations.

[5] See Barbier and Markandya (1990) for similar notion for sustainable economic growth.

[6] Since the system of equations (5)–(7) is a system of continuous autonomous differential equations there exists a unique solution for each set of initial values. The solution for emissions, x, including initial level of emissions constitutes an optimal control for dynamic optimization in the absence of stock constraints.